Differentiation and mediation: An investigation in early-grade mathematics pedagogy

by

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COMPULSORY DECLARATION

This work has not been previously submitted in whole, or in part, for the award of any degree. It is my own work. Each significant contribution to, and quotation in, this dissertation from the work, or works, of other people has been attributed, and has been cited and referenced.

Signature: Signed by candidate   Date: 7 February 2020

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Abstract

Differentiation and mediation: An investigation in early-grade mathematics pedagogy

This thesis explores differentiating practices within the context of an early-grade mathematics intervention. Through qualitative research, the study looks at pedagogic interpretations of differentiated instruction. Situating mediation as a central feature to differentiated instruction, this thesis investigates two key features. Firstly, mediation requires differentiating in terms of grouping – in both cognitive development and classroom organisation. Secondly, mediation requires consideration of the form of pedagogy between teacher and learners. In an effort to consider mediation systematically, I explore the productivity of a Bernsteinian framework through which the data can be measured. Through control relations in the selection, sequence, pace, evaluative criteria and hierarchical rules within the pedagogic structure, my intention is to illustrate how mediation 'happens' in different ways.

Locating this study within three Grade 3 classes in the same school, the analysis reveals a trajectory in the pedagogic structure across tasks within lessons. Certain patterns emerge in the data and, with mixed control relations in evaluative criteria and weakened control of hierarchical rules in particular, a framework is developed with which mediation can be measured. Ultimately, the analysis provides a micro-lens into mediation within a differentiating pedagogy and a framework through which mediational patterns might be explored in a larger or more varied sample.

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Chapter 1: Introduction

According to recent statistics, the teacher to learner ratio in South African classrooms is 1:35. A more candid account is that one in four schools are faced with more than 45 learners per teacher, making effective teaching and learning problematic. Not only are classes large, but "contemporary student populations are becoming increasingly academically diverse" (Singh, 2014, p. 58), making every class effectively a 'multi-grade' class. Faced with a spectrum in levels of development, a whole-class or one-size-fits-all teaching approach, particularly in mathematics, is neither productive nor efficient. Making decisions on the instruction of mathematics, one should then first consider whether transmission and acquisition should be a mere formality of rules and procedures, or whether it be a sense-making journey towards conceptual understanding. In support of the latter, one must consider that individual needs may differ and whole-class teaching can only benefit some. Through differentiated instruction, there is the potential to meet learners at current levels of development and build on their existing knowledge.

Over the years, differentiated instruction has evolved and taken on many forms, proving effective on the one hand, while limiting opportunities on the other. While often carrying with it influences of race, gender, culture, language, and socio-economic status, if one peels away the negative biases and focuses rather on its potential for effective teaching and learning, one can begin to explore the role differentiation plays in mathematics instruction. With differentiated teaching, the intention is to optimise learning and improve overall achievement in early-grade mathematics. By mediating instruction within the levels of understanding of the individual - matching concepts to learner's actual stages of cognitive development and pacing instruction accordingly – the objective is that learners will be able to benefit more effectively.

Accepting then that teachers *do* differentiate instruction, one needs to consider teachers' understandings and applications of such a pedagogy. With this comes the responsibility of the teacher when selecting, sequencing and pacing content in ways most conducive to learning with understanding. The importance of 'mathematical knowledge for teaching' (Ball, Hill & Bass, 2005) comes into play in that it is not only teachers' knowing mathematics, but being able to transfer such knowledge through a logical progression in ways that learners can grasp and apply meaningfully. How the teacher groups learners according to levels of development and then pitches content conceptually appropriate to these levels is what shapes differentiated instruction and its potential for success.

Focal research problem

This thesis aims to explore the practices of differentiated instruction within the context of early-grade mathematics. Through qualitative research, the analysis considers three mathematics lessons by three different teachers within a Grade 3 public school in South Africa. This school participates in the *NumberSense Mathematics Programme.* The programme provides teachers with weekly classroom-based support in differentiated teaching and learning. With focus on the interactional relationship between teacher and taught, my thesis places mediation as a central feature to differentiated instruction. Through analysis of control relations between the teacher and learners in the pedagogic structure, the aim is to develop a model for investigating differentiation and the role of mediation in early-grade mathematics.

By combining Vygotsky's *socio-cultural theory* (1978) with Bernstein's framing of *pedagogic discourse* (1990), I investigate mediational practices within a differentiating pedagogy. Drawing on empirical studies of Hoadley (2006), Morais, Neves and Pires (2004) and Pausigere (2016) who use adaptations of Bernstein's coding scheme in search of the 'ideal' pedagogy, I look at control relations between the teacher and learners and suggest a framework through which differentiation in mathematics can be measured.

My main research question for this thesis is:

What are the practices of differentiated instruction within the context of an early-grade mathematics intervention?

Secondary questions consider:

- What are the pedagogic approaches toward differentiated teaching and learning in early-grade mathematics?
- What are the tools and justification for the grouping of learners in differentiated instruction?

- What is the role of mediation in differentiated instruction?
- How productive is a Bernsteinian framework in measuring control relations within the context of differentiated instruction?

Outline of the study

Through an investigation of differentiation and mediation in early-grade mathematics pedagogy, this thesis has been organised in the following way.

This first chapter describes the rationale for the study, introducing its purpose and related research questions. Chapter 2 provides a literature review of empirical research in the field of mediation and differentiated instruction. The review follows with research into existing frameworks for mediation and pedagogies effective to teaching and learning. Chapter 3 provides a theoretical framework derived from the work of Basil Bernstein and Lev Vygotsky in developing a model through which mediation in differentiation can be measured. In Chapter 4, I outline the research design behind the data analysis. This describes the mathematics intervention programme and includes the frameworks through which the data is analysed. Chapter 5 provides a detailed analysis of the data, in terms of the grouping and the pedagogic structure within three Grade 3 mathematics lessons. Finally, Chapter 6 presents a discussion around the data analysed and draws points of interest from the results and findings in the data sample. The thesis ends with a summary and presents the implications and limitations of the study.

Chapter 2: Literature review

2.1 Introduction

Through broad inquiry into existing research on differentiated instruction, one comes to realise the many forms it takes and the mixed conceptions attached to the notion. Firstly, it seems necessary to *define* differentiated instruction (DI), for the purposes of this thesis, and in relation to other forms of grouping when teaching and learning mathematics. Secondly, there is the need to *justify* differentiation on the basis that learners should understand and make sense of mathematics. To accomplish this, teachers also require an understanding of learners and their thought processes in mathematics. With this, teachers need to meet learners at the level of development most suited to each individual. Thirdly, if one *is* to differentiate, one needs to *select* the necessary path (pedagogy) in leading the individual towards a progressive conceptualisation (framing) of mathematics. With different framing for different individuals, teachers can better assist in the construction of new concepts and content at levels developmentally appropriate. Put simply then, by *defining*, *justifying* and *selecting* methodologies towards DI, the *what*, *why* and *how* of DI, my intention is to contribute towards an understanding of such a pedagogy and develop a framework through which it can be measured.

The literature review which follows is therefore structured in this way:

- What is differentiated instruction?
- Why differentiate instruction?
- How to differentiate instruction.

Following the discussion of *what*, *why* and *how*, the literature review looks toward a measure for mediation. In search of a framework through which mediation can be measured, the review turns to pedagogic forms effective to teaching and learning. This considers research in control relations between the teacher and learners and how mediation within a differentiating pedagogy can be explored.

2.2 The what, why and how of differentiated instruction

2.2.1 What is differentiated instruction?

In the attempts to accommodate heterogeneity in the classroom, and in mathematics in particular, a multitude of instructional methods have woven their way through schools and classrooms over the years, each with their own sets of merit and complication. In broad terms, justification for grouping is on the basis of "the need to adapt content, pace and teaching methods to students functioning on different levels" (Dar, 1985; Slavin, 1988; Sørensen & Hallinan, 1986; cited in Linchevski & Kutscher, 1998). This depends largely on the nature of the subject. In the case of mathematics, with content largely progressive and hierarchical in structure, it makes sense to work at and accommodate different levels.

Empirically, researchers such as Slavin and Karweit (1985), Sørensen and Hallinan (1986), Subban (2006), and Linchevski (1998) have conducted a number of studies on varying forms of differentiation, in seeking an 'ideal' pedagogy and improving academic achievement. Comparing combinations such as streaming, tracking, within-class ability grouping, individual instruction, same-ability grouping and mixed-ability grouping, such researchers, in their own design, consider the results and long-term effects of these dynamics based on learner achievement before and after initiating particular methodologies. All ask questions such as:

- Does ability grouping succeed in improving the achievements of learners in general and the weaker ones in particular?
- What are the effects of whole-class, ability-grouping, and individualised instruction on mathematics achievement?
- What are the effects of teaching mathematics in a mixed-ability setting on learner's achievements and teachers attitudes?
- Will differentiation completely meet the complex needs of all heterogeneous learners in the regular classroom?
- Can inequality be avoided?

While studies conclude with mixed findings around the various forms of differentiated instruction, Sørensen and Hallinan (1986) take a slightly different approach by zooming in to the effects of *within-class* grouping rather than comparisons between different forms of grouping. Attention is drawn to the fact that, through within-class grouping, there is increased attention in that by “adapting the instructional material to the aptitudes and preparation of students (it) helps students to make better use of their abilities” (Sørensen & Hallinan, 1986, p. 522). Through research, they also found that the use of time in homogeneous grouping was more constructively used, “allowing the teacher to cover more material in the same time period and thus provide more opportunities for learning for students” (ibid.).

Acknowledging the lack of theoretical support in the field, Subban (2006) creates a research basis with connections to Vygotsky’s *socio-cultural theory of learning* (1978). Here the focus is drawn to the idea that “the areas of social interaction, engagement between teacher and student, physical space and arrangement, meaningful instruction, scaffolding, student ability and powerful content all become elements to consider within the context of contemporary education” (Subban, 2006, p. 937). Drawing on Vygotsky’s *Zone of Proximal Development* (1978) and the power of language and speech as tools toward higher mental functioning, Subban highlights the role of *mediation* between teacher and learner. Subban also comments that “ignoring these fundamental differences may result in some students falling behind, losing motivation and failing to succeed” (Ibid., p. 938). Subban acknowledges the need for teachers to accommodate different levels of readiness and shift thinking from merely ‘completing the curriculum’ toward ways in which teachers can bridge the gap between where learners are developmentally, and where they need to be. Most importantly though, Subban recognises that DI allows the teacher to focus on the same key principles for all learners, whatever the instructional process. What differs, however, is the pace and varying levels at which this process takes place.

Thus far then, presented with a range of terms pertaining to differentiated instruction – streaming, tracking, ability-grouping, etc. – it seems fitting to pause and clarify what is meant by DI before delving deeper into justification of such an approach.

Differentiated instruction, for the purposes of this thesis, is best defined as the process of “ensuring that what a student learns, how he or she learns it, and how the student demonstrates what he or she has learned is a match for that student's readiness level” (Tomlinson, 2004, p. 188). Recognising differences in background knowledge, differentiation is on the basis of learner’s levels of cognitive development at a given point in time, providing guided instruction where understanding is achievable and steering learners toward higher order thinking. DI refers to grouped instruction *within* the classroom, rather than separating learners into different classes or schools. By grouping learners differentially within a mathematics lesson, the intention is to place together learners of similar levels of development, a homogeneous grouping, where they can learn most effectively.

2.2.2 Why differentiate instruction?

> Our analysis of the development of children's mathematical thinking can be thought of as scientific knowledge, as defined by Vygotsky (1962), that provides a basis for teachers to interpret, transform, and reframe their informal or spontaneous knowledge about students' mathematical thinking.
>
> (Carpenter, Franke & Fennema, 1996, p. 5).

Through research on cognitively-guided instruction, Carpenter, Franke and Fennema (1996) build on the knowledge of learners and teachers as a means of better understanding thought processes. By developing a programme to help teachers understand the way in which learners think, it is argued that teachers can make better instructional decisions. While it may seem obvious to consider learners' understanding, research shows that there is little consideration of such when making instructional decisions in the classroom. By identifying what learners already know, and building on existing knowledge, the teacher can better facilitate learners in 'constructing' mathematical knowledge rather than merely transmitting it to them. Through a suggested problem-driven approach, and careful *mediation*, the teacher can lead learners toward higher levels of development or higher thought processes. By moving from problems which learners *can* solve and assisting them towards higher order functions, learners make meaning and sense of what they are doing. It is argued that there is, logically, no point in teaching learners the mere content to be covered if they have not taken the necessary conceptual steps from where they are towards where they need to be.

In a similar spirit, Venkat (2013) refers to 'temporal range' as the process of "building new process layers on previous processes" (Venkat, 2013, p. 32). She refers to two dimensions – mathematics within a time and space and, the learner's level within mathematics. With the two bisecting at different points, Venkat argues that skilful mediation requires the teacher to be able to identify these different 'junctures' within a class and provide a strategy in moving forward toward higher levels of conceptual development. The teacher needs a clear understanding of the progression and the trajectory of mathematics in guiding learners toward higher levels of abstraction, as well as a clear indication as to individual development thus far. Venkat argues that the South African curriculum places too much emphasis on grade-to-grade progression and pacing, without taking into consideration the past and future of learners mathematical knowledge. By focusing more attention on existing capabilities, teachers can better support learners progressively and sequentially on a road toward conceptual understanding rather than merely 'covering' the curriculum. "Looking at poorly paced and sequenced teaching in terms of temporal range indicates a lack of awareness of the nature of mathematical progression or, perhaps, a reluctance to push forwards into more complex mathematical terrain within teaching" (Ibid.). By teaching differentially, one needs to consider the learner's current level of understanding while at the same time having the sufficient knowledge of the curriculum (selection). This includes the progression of mathematical concepts (sequence) as well the time available to not only cover the curriculum but also work at a speed (pace) accessible to the various differentiated groups.

Venkat and Naidoo (2012) take this one step further in *Analyzing coherence for conceptual learning in a Grade 2 numeracy lesson*. Building on previous research, the paper "presents evidence to illustrate, firstly, poor coherence in and across pedagogic communication and activities and secondly, random selection and sequencing of exercises that militate against meaning making" (Venkat & Naidoo, 2012, p. 21). Venkat and Naidoo define coherence not only in terms of parts which fit together but also the mathematical sense made by learners within these parts. Coherence requires continuity and "a key aspect of continuity relates to connections between later and earlier parts of a text – between 'given' and 'new' information (Chafe, 1976, cited Venkat & Naidoo, 2012, p. 22). Through analysis of a series of mathematics lessons, Venkat and Naidoo recognise a lack of direction or connection in lessons and between tasks. Noticing random selection and sequencing of tasks and exercises, learners were unable to make the necessary connections between existing and expected knowledge structures. Failing to communicate concepts coherently and without making connections to previous concepts, teachers marginalise the possibility for sense-making. Aimed at improving teaching and learning of mathematics, this study highlights the fundamental role of the teacher in transmitting tasks and activities with a coherent, natural progression which builds on existing understanding.

2.2.3 How to differentiate instruction.

Common thus far is cognisance of the interactional relations between teacher and taught, varying levels of understanding within classes and the need to adjust pedagogy according to individual development.

With specific interest in these interactions, Hasan (2002 & 2004) focuses on semiotic mediation and the role it plays in the development of higher mental function. Where 'semiotic' refers to acts of meaning, Hasan (like Vygotsky) attaches most significance to the act of language. Hasan believes that considering the interactive exchange between people, their relation to one another and in relation to the 'object' of interaction, is the point at which meaning is construed. Where there is language, there is mediation and it is through this process that higher mental function evolves. Defined more simply, Hasan states that:

> "the noun 'mediation' is derived from the verb 'mediate', which refers to a process with a complex semantic structure involving the following participants and circumstance that are potentially relevant to this process:
>
> [1] someone who mediates, ie a mediator;
> [2] something that is mediated; ie a content/force/energy released by mediation;
> [3] someone/something subjected to mediation; ie the 'mediatee' to whom/which mediation makes some difference;
> [4] the circumstances for mediation; viz,.
> (a) the means of mediation ie modality;
> (b) the location ie site in which mediation might occur."
>
> (Hasan, 2002, p. 4)

Defined as a logically ordered process, Hasan also relates an analytical perspective in better characterising semiotic mediation:

> verbal interaction → meanings construing experience → experience construing mind
>
> (Hasan, 2004, p. 34)

Through the process of interaction (by means of language), meaning is translated into experience. This experience leads to higher mental function.

Drawing from theory of Vygotsky (1978) and Bernstein (1990), Hasan (2002 & 2004) explores the role of mediation in the acquisition of specialised knowledge. According to Hasan (2002), mental development stems from semiotic mediation. With specific reference to language, Hasan talks of the relationship between mental function and social activity, emphasising that "true direction of mental development is not from the individual to the social, but from the social to the individual" (Hasan, 2002, p. 5). With mediation comes sociality, and it is through this two-way process that teachers can guide learners toward higher order thinking.

Hasan (2004) further clarifies by saying that concept formation is not a passive process, but rather requires participation, a sense of engagement and interaction with other learners. This creates the basis upon which mediation takes place and provides the potential for acquiring specialised knowledge. It is therefore the teacher's role in drawing learners' attention to the point of engagement and negotiation. From here, the teacher also needs to ask relevant questions, articulate answers and facilitate discussion and reflection in ways that lead to higher order thinking.

With much of the control placed with the teacher, it becomes obvious that "different forms of semiotic mediation entail different forms of higher mental function" (Hasan, 2004, p. 34), which creates the potential for (or risk of) differential access to higher order thinking.

> "the fact that language always mediates does not mean that it necessarily mediates what someone set out to mediate; if that were the case there would be no educational failures. What gets mediated depends a great deal on the mental disposition of the addressee. Thus something gets construed through meaning; successful mediation in pedagogic sites is that where the intended approximates the achieved." (Hasan, 2004, p. 41)

With Hasan as the platform from which the role of mediation emerges in my research, what follows is further inquiry into mediation, its role in teaching and learning and specifically in differentiated instruction.

For Shabani (2010), "teaching of a student (is) not just as a source of information to be assimilated but a lever with which the student's thought, with its structural characteristics, is shifted from level to level" (Verenikina, 2003, cited in Shabani, 2010, p. 239). Shabani looks towards Vygotsky's *socio-cultural theory* (1978) and the implications for instruction for both teachers and learners. Social interaction sits at the heart of this theory, and with language as the vehicle of thought, it forms the basis of learning and development. Shabani argues that "learning as a mediated process is social in origin and then becomes individual as a result of linguistically mediated interaction between the child and the more experienced members of the society" (Shabani, 2016, p. 2). He also considers Vygotsky's *Zone of Proximal Development* (ZPD) as the space where learning occurs. By situating learners within their ZPD's, and by working through shared activities and understandings, social mediation triggers cognitive development and moves learners towards higher levels of capability. He emphasises the importance that learners are placed within their ZPD's and that social interactions be framed within tasks or activities that have a clear purpose in mind.

In situating learners correctly, Shabani (2010) also highlights the importance of accurate assessment to diagnose learners' ZPD's. He claims that "true diagnosis must provide an explanation, prediction, and scientific basis for practical prescription" (Shabani, 2010, p. 239). Shabani goes a step further in suggesting that Vygotsky's ZPD is also applicable to adults, with the potential to shift teachers from current levels of development toward higher mental functions. This holds the potential for lifelong change in teachers' ZPD's and better equips them to understand and act on varying levels of development of learners within their own classes.

Similarly, Bartolini and Mariotti (2008) consider the communication channel between teacher and learners through semiotic mediation in the mathematics classroom. Drawing on Vygotsky's *socio-cultural theory* they argue that, through the ZPD and the process of internalisation, cognitive development takes place. With the ZPD defined as the distance between actual and potential developmental levels and internalisation defined as "the internal reconstruction of an external operation" (Vygotsky, 1978, cited in Bartolini & Mariotti, 2008, p. 7), these processes intersect and individual knowledge is constructed through social experience. Bartolini and Mariotti state that it is through this semiotic, collaborated process or co-construction of knowledge between teacher and learner, that the individual moves toward higher mental function. Bartolini and Mariotti also emphasise the crucial role of the teacher "as the tool for semiotic mediation and the ultimate witness responsible for the meaning to be constructed and appropriated by students" (Bartolini & Mariotti, 2008, p. 778). For the most part, the teacher holds the power and control in the extent to which cognitive development evolves.

Nyikos and Hashimoto (1997) take mediation a step further by suggesting a 'group ZPD' whereby learners are grouped according to similar levels of development. In arguing the benefits of such a pedagogy, it "debated the possibility that the potential growth for the group as a whole can be pointed to where the individual ZPD's intersect and that through the social mediation of the group's interrelated ideas, the group ZPD can grow exponentially" (Schmitt, 2009, p. 36). By identifying a point of intersection, a zone of potential growth for the whole group, one can develop and move toward higher levels of cognitive demand. Through collaborative interaction, multiple discussions and points of view, the group moves together from current capabilities to higher levels of development.

Goos, Galbraith and Renshaw (2002) concurred by arguing that "each learner requires the contribution of another's skill and knowledge to reach greater learning potential" (Goos, Galbraith & Renshaw, 2002, p. 195). By marking individual levels of development and grouping learners into a collaborative ZPD one can, through mediation, facilitate learning within the zone of proximal development. What remains important is the "teacher's role in orchestrating students" discussions and social interactions (Ibid., p. 194) in ways that are constructive to development. With 'collaboration' as the "coordinated, synchronous activity that is the result of a continued attempt to construct and maintain a shared conception of the problem (Teasley & Roschelle, 1993, cited in Goos, Galbraith & Renshaw, 2002, p. 235), it is argued that these interactions should be more than simply agreeing or disagreeing to an answer but rather a complex process of co-constructed understanding. Concepts should also be introduced explicitly and systematically if learners are to achieve individual higher order thinking.

From a more critical point of view, while Shotter (1993) acknowledges Vygotsky's (1978) concept of social mediation, he also points to certain flaws or ambiguities in its practical application. While Vygotsky argues for social interaction through language, Shotter argues that what he doesn't make clear is "how the child learns in what way a task must be done and what way mistakes must be corrected" (Shotter, 1993, p. 68). Shotter makes the point that while Vygotsky's theory holds true and has made invaluable contributions to research, there is a 'gap' in the theory in that the application and success lies very much in the hands of the teacher. Shotter argues that the "teachers are the keepers of the culture that the child must acquire" (Ibid., p. 69).

2.3 Measuring mediation

Drawing on Vygotsky's (1978) theory on social mediation and the role of interaction in teaching and learning, my intention was to make use of an existing framework with which the efficacy of such interactions could be measured. In search of such a framework, inquiry took the path of researchers such as Pol, Volman and Beishuizen (2010), Hennessy et al. (2016), Hermkes, Mach & Minnameier (2018) and Schoenfeld (2017). Among others, these studies have sought to create a framework for mediation, or social interaction or classroom dialogue. In line with Vygotsky's theory, each suggest similar frameworks where, by first identifying individuals' ZPD's, the teacher communicates (or mediates) in a language suited to particular groups and provides better support in the development towards higher cognitive function. Beginning with a diagnosis of levels of understanding (learner's ZPD's), it is argued that teachers can facilitate and guide learners within a level of appropriate cognitive demand.

Pol, Volman and Beishuizen (2010) make connections between mediation and scaffolding in creating a framework that measures the effects of classroom interaction. They look towards a temporary structure or support provided by the teacher to the learner which aids in further development understanding. Hennessy et al. (2016) develop a coding scheme as a framework which analyses classroom dialogue. Placing language as a central feature for thinking and a mediator of activity, it is argued that it allows for the co-construction of knowledge and meanings. Through video, Hermkes, Mach and Minnameier (2018) provide a methodology which measures scaffolding in small group class settings where, through tasks and sequences, learners are coded according to levels of understanding. Schoenfeld (2017) also turns to video as a tool to understand and improve mathematical thinking and teaching. He argues that looking at classroom interactions provides teachers with an inside view on how to review and better plan future instruction. Schoenfeld's study provides 'lenses' through which teachers and learners can analyse and improve on teaching and learning of mathematics.

Having reviewed the literature however, each of these studies show that mediation is a phenomenon difficult to measure. As mediation is not fixed or pre-determined but rather something that varies between individuals or groups at any given point, each of these studies conclude with limitations and suggestions for further development.

Returning then to Shotter's statement that "teachers are the keepers of the culture that the child must acquire" (Shotter, 1993, p. 69), I came to consider the role and impact of interactional and control relations between the teacher and learners. Due to the temporal nature of mediation and rather than defining mediation through a fixed framework, is the suggestion of a pedagogy through which mediational patterns can be measured. This relates to studies of Hoadley (2003 & 2006), Morais (2002), Morais, Neves and Pires (2004) and Pausigere (2016) who look towards the conditions necessary for successful transmission and acquisition of knowledge. Ultimately, the study returns to Hasan who, through Vygotsky's *semiotic mediation* (1978), turns to Bernstein's framing of the *pedagogic discourse* (1990) as a measure for mediation.

Drawing on Bernstein's concepts of classification and framing, Hoadley (2006) develops a framework for the analysis of pedagogic variation. Hoadley provides a language for the description of pedagogy and its variation in transmitting 'a school code' or specialised knowledge. Hoadley suggests that framing "opens up the potential for the change of boundaries, the contesting of power relations. It is through interaction (framing) that boundaries between discourses, spaces and subjects are defined, maintained and changed" (Hoadley, 2006, p. 4). By considering such interaction then, the focus is on the relationship between teacher and learner and the "importance attributed to language as a mediator of the development of higher mental processes" (Morais, Neves & Pires, 2004, cited in Hoadley 2006, p. 15) that lead to specialised knowledge. Hoadley makes particular reference to the explication of evaluative criteria in making clear to learners the expectations and requirements for the successful completion of a task. This requires "clearly telling children what is expected of them, identifying what is missing from their textual production, clarifying the concepts, leading them to make synthesis and broaden concepts and considering the importance attributed to language as a mediator of the development of higher mental processes" (ibid.). Through this mediational process, it is argued that learners can better move toward higher mental function.

In earlier work, Hoadley (2003) hones in on Bernstein's notion of framing (1990), looking specifically at pacing and the role it plays in the production or reproduction of social relations within the pedagogic structure. With framing as the degree of control teacher and learner possess over knowledge transmitted and received, Hoadley refers specifically to two aspects of pacing: the pacing rules of different transmission practices and differentiating between different student learning rates. Hoadley (2003) also presents the distinction between external and internal framing. While internal framing refers to the pedagogic relationship between teacher and learner, external framing refers to the relationship/control between the teacher and that of the school, curriculum and policy. With strong and weak values for each, both refer to the level of power and control of the teacher in terms of the pacing of instruction. Constrained by the rules of curriculum and policy, a common complaint in South African schools, teachers are often left with little flexibility in weakening the frames of pacing in the race to 'complete the curriculum'. Given the freedom/control over pacing however, teachers could adapt the rate of instruction differentially, "to the point at which learning is expected to occur" (Hoadley, 2003, p. 267). Such is the nature that internal framing is embedded within the social relations of external framing.

"Bernstein, discussing the rules of transmission, or 'pedagogic relay', refers to pacing as 'the expected rate of acquisition, that is, the rate at which learning is expected to occur… Pacing rules, then, regulate the rhythm of the transmission, and this rhythm may vary in speed" (Hoadley, 2003, p. 272). While selection and sequence are often contained in curriculum specifications, the real skill lies in the teacher's success in fine-tuning pacing to suit different levels of development within the mathematics class. Hoadley presents this in a case study where by comparing two schools, a working class and a middle-class school, she contrasts the degrees of control over pacing. Through consideration of both external and internal control relations and their significance to one another, Hoadley concludes her study by considering the ways in which "instructional practices are located within specific sets of social relations which potentially are a significant factor in the regulation of teachers' practices in the classroom" (ibid.).

Morais, Neves and Pires (2004) explore a sociological theory of transmission and acquisition. Based on Bernstein's *pedagogic discourse* (1990), they discuss the importance of a mixed pedagogical practice in attaining the recognition and realisation rules necessary for certain concepts within a subject. This relates to a mixture of strong and weak classification and framing in the transmission and acquisition of knowledge. In their study on *The what and how of teaching and learning*, Morais, Neves and Pires describe the search for the "characteristics of pedagogic practices most favourable to the acquisition of (scientific) knowledge" (Morais, Neves & Pires, 2004, p. 75). While their focus is on scientific understanding, this can also be applied to mathematics as both subjects entail progressive and cumulative conceptual acquisition. Through the theory of Vygotsky (1978) and Bernstein (1990), Morais, Neves and

Pires (2004) suggest a pedagogy that is mixed. This requires a shift in control relations between the teacher and learners and teaching at a level of conceptual demand slightly higher than learners' actual development. By applying an investigative approach such as problem solving, it is argued that it allows *all* learners the potential to develop complex cognitive competences. The study emphasises the crucial role of interaction and that the shift in power and control relations between teacher and learner provides the potential for cognitive development.

By understanding and identifying differential achievement in a subject such as science (and mathematics), one then needs to analyse the discourse and the rules that regulate transmission and acquisition. Morais, Neves and Pires (2004) turn to Bernstein's (1990) regulative and instructional discourse as well as classification and framing as a means of decoding the pedagogic discourse. In so doing, careful consideration is taken in terms of *what* it is that needs to be transmitted and acquired and *how* this is best achieved. Morais, Neves and Pires illustrate how, through mixed pedagogy - combinations of strong and weak classification and framing - learners gain the necessary access to and understanding of specific concepts. By allowing flexibility over the pacing of a lesson, for example, learners have a better chance of successfully acquiring the concepts/skills before moving on to higher order concepts. While framing can be weakened, it is also necessary to maintain stronger classification in terms of content in that "there are knowledges and competences of a high order to be learned by all children and the school should make them available to all" (Morais, 2002, p. 561). More specifically, Morais, Neves and Pires argue for the explication of evaluative criteria as "a crucial condition for efficient scientific learning" (Morais, Neves & Pires, 2004, p. 85). For explication to be successful, it requires open communication relations and if learners are required to be explicit in their productions, it should be through interaction with others. Framing over evaluative criteria should therefore be strong while hierarchical rules remain weak. With explication and elaboration comes time and while this indicates weakened framing over pacing, Morais, Neves and Pires (2004) argue that it is not the physical time required but rather the greater allocation of time for active engagement in investigative tasks.

This relates back to a key aspect of differentiated instruction in that, while structuring of time may vary between groups, there is still the responsibility of the teacher in making sure the necessary grade-appropriate content is covered.

Based on Bernstein's (1990) sociology of pedagogy and aligned with Hoadley (2003 & 2006) and Morais, Neves and Pires (2004), Pausigere (2016) proposes a mixed pedagogy in the attempts to interrupt social reproduction. Through Bernstein's framing component in particular, Pausigere argues that mixed pedagogy can overcome social backgrounds and inequalities. With framing as the degree of control in "the selection, sequencing, pacing and criterial rules of the pedagogic communicative relationship between transmitters/acquirer(s)" (Bernstein, 1990, p. 204), adjusting control relations provides all learners with equal opportunity to educational knowledge. Pausigere draws attention to the role of classroom communication, interaction and the relationship between teachers and learners.

With emphasis on pacing, Pausigere (2016) relates to studies which show how strong pacing limits disadvantaged learners while relaxed pacing enhances the possibility for acquisition and engagement. "Responsive pacing considers learners social position, context, and needs and, most importantly, their levels of understanding" (Pausigere, 2016, p. 48). While Pausigere argues for relaxed pacing, he also emphasises that this should not limit or restrict curriculum coverage but rather requires the careful planning and consideration of the teacher. A knowledgeable teacher will be equipped in ensuring that the core mathematical concepts are covered. By acknowledging the individual through mixed pedagogy, "such insights illustrate the critical features of a humanising pedagogy, instigate social change within primary maths education, and increase access to mathematics learning for all children" (Ibid., p. 51).

Drawing from such research of Hoadley (2003 & 2006), Morais, Neves and Pires (2004) and Pausigere (2016), my intention is look to Bernstein's framing of pedagogic discourse as a measure for mediation. By considering control relations between the teacher and learners, I suggest a pedagogic framework through which mediation within differentiation can be explored.

2.4 Conclusion

In this chapter, I have set out to review the literature of differentiated instruction. By first defining *what* it means to differentiate, the intention is to recognise its diversity and to clarify what it means for the purposes of this study. Second, by describing *why* it is important, the objective is to make a positive claim for differentiation through consideration of the individual in the classroom setting. By acknowledging varying levels of development and guiding instruction to suit individual needs, the teacher can effectively build on existing knowledge structures and lead learners toward higher competencies. Third, is the suggestion of *how* differentiated instruction functions. By proposing mediation as a fundamental feature in the transfer of knowledge, it is argued that the structured interactions between teacher and learners lead to higher mental function.

While mediation is difficult to measure, I look towards features of pedagogy effective to teaching and learning. Drawing on Hasan's theoretical connections between Vygotsky (1978) and Bernstein (1990), the intention is to measure semiotic mediation through Bernstein's coding scheme of pedagogic discourse in search of a pedagogy optimal to mathematical proficiency. Put differently, this thesis aims to make use of Bernstein's theoretical language as a resource for describing mediation toward an 'ideal' pedagogy. This is expressed in greater detail in the theoretical framework which follows.

Chapter 3: Theoretical framework

3.1 Introduction

In this chapter, I return to Hasan (2002 & 2004) who makes connections between Vygotsky (1978) and Bernstein (1990) in researching the concept of semiotic mediation. Drawing on Vygotsky's *socio-cultural theory of mediation* as a central feature to differentiated instruction and situating learners within their *Zone of Proximal Development* (ZPD), I aim to show how differentiating pedagogies reveal themselves in different forms. With Bernstein's *pedagogic discourse*, I also illustrate how varied control relations provide a language for describing mediation. By describing relations between these theorists is the suggestion of a framework through which mediation can be measured. With Hasan as the point of departure, this chapter provides a more detailed description into the theories of Vygotsky and Bernstein and how they are applied in the context of differentiated instruction.

3.2 A Vygotsky-Bernstein synthesis of mediation

Based on studies around Vygotsky's (1978) semiotic mediation, Hasan's interest lies in how consciousness is formed.

> "How consciousness is formed; how its distribution carries in form across different classes and groups in a society; what institutions contribute, and how, to such distribution; and what part variation in consciousness plays in the production and reproduction of society are all issues of importance to such a theory of sociology"
>
> (Hasan, 2004, p. 30)

With the belief that consciousness is central to any code-based theory of sociology, Hasan (2004) makes connections between Bernstein (1990) and Vygotsky (1978) and provides the suggestion of a framework through which mediation can be measured. Hasan argues that Bernstein's 'take' on consciousness is of much relevance to Vygotsky's theory and while he does not specifically use the term 'semiotic mediation', Hasan believes social interaction to be essential to the formation of consciousness. Like Vygotsky, Bernstein attaches a great deal of importance to language, how it is used and how it "comes to reorganize cognitive abilities"

(Shotter & Lock, 2012, p. 63). Where they differ, however, is where Vygotsky sees homogeneity in social relations, Bernstein sees heterogeneity. Where Vygotsky sees social interaction, through language, as central to mental development, Bernstein focuses more specifically on the marked differences created through these social relations.

Vygotsky speaks of two lines of human mental activity, a natural line and a social line. While the natural line relates to basic mental functions, the social line leads to higher mental functions. Important for Vygotsky is that these two lines are interlinked and that through language, thought processes can be internalised. Hasan connects this to Bernstein's "mediational power of language" where "speech systems or codes create for their speakers different orders of relevance and relation" (Hasan, 2002, p. 10). Bernstein believes that the production and reproduction of society is not possible without 'social subjects' and that "forms of social relation act selectively upon what is said, when it is said, and how it is said… (they) can generate very different speech systems or codes… (which) create for their speakers different orders of relevance and relation" (Bernstein, 1971, cited in Hasan, 2004, p. 36). While this often comes with negative or privileged implications, the intention for this thesis is that "different speech systems or codes" create the opportunity for differentiated instruction with the intention of reaching all levels of cognitive demand.

For Bernstein, code regulates relationships between contexts and the communication within these relationships. By being able to regulate interaction, one can distinguish between different forms of mediation. Bernstein's codes are known through classification and framing and are measured in terms of strength or weakness. "Since strength and weaknesses are two end points of a continuum, in theory, variation can yield a large array of modes of mediation, particularly when applied to framing, which is itself a cluster of aspects of communicative practice each of which may vary independently" (Hasan, 2004, p. 39). It is through these classification and framing values that Hasan suggests a framework through which mediation can be measured.

Drawing on Hasan's connections, what follows is a closer account into Vygotsky's *socio-cultural theory* on mediation and Bernstein's control relations in *pedagogic discourse* to be used as a framework through which differentiated instruction is measured.

3.3 Lev Vygotsky

"Mediation is a Vygotskian concept that originated in Russia in the early 1900s and it explains the process of instruction in the ZPD" (Schmitt, 2009, p. 25). By combining aspects of his *socio-cultural theory* and *semiotic mediation* through language, it is possible to see how, in a sense, Vygotsky "paved the road for differentiated instruction" (Essays, 2013). At the heart of Vygotsky's interest lay the means by which learners could gain access to theoretical knowledge in all its forms – a knowledge separate from the everyday.

With semiotics as the study of meaning-making or meaningful communication, Vygotsky (1978) emphasises the importance of language as a mediating factor in accessing theoretical knowledge. With language as the vehicle of thought, Vygotsky describes that "human mental activity is a *mediated* process in which symbolic and socio-culturally constructed artifacts, the most significant of which being language, play an essential role in the mental life of the individual" (Vygotsky, 1978, cited in Shabani, 2016). Vygotsky sees language and speech as tools to mediate knowledge and develop human consciousness, with humans as the tools through which mediation is made possible. He describes semiotic mediation in both *visible* and *invisible* terms.

- Visible mediation is deliberate and involves the clear transmission and acquisition of a specific concept or skill between the teacher and learner(s). There is a good understanding of the goal to be achieved and there is active participation from both parties.

- Invisible mediation is implicit in its intentions. Learners are not aware of the teaching or the learning taking place, especially the underlying concepts or skills. "The interactants do not 'see' what is being mediated; what they 'see' is some process of everyday living" (Hasan, 2002, p. 114). Much weight thus lies in the role and influence of the teacher in framing instruction to suit individual needs and develop higher order thinking.

In accessing theoretical (scientific) knowledge, Vygotsky's development of theory is based "on the premise that the individual learner must be studied within a particular social and cultural context" (Subban, 2006, p. 936). Of primary concern is the means by which teachers assist learners towards higher order functions, that which can only be acquired through social interaction. Social interaction then plays a key role in the development of cognition. This is, however, "not a simple one-way process of transmission but a complex pedagogic process in which a learner's every day concepts are extended and transformed by theoretical concepts" (Young, 2009, p. 200). Social interaction relies on the transfer of knowledge from 'the other' towards a 'self-regulated' knowledge (from the teacher to the learner). Hasan elaborates by describing this relationship as "a complex semantic structure involving the participants and circumstance that are potentially relevant to the process" (Hasan, 2005). By considering the mediator, the mediatee, the content mediated, as well as the circumstances or means for mediation within a school environment, Hasan argues that learners gain access to higher order thinking. The teacher (adult/expert/more knowledgeable) plays a key role in guiding the learner through this process. By modifying content and adjusting levels of assistance to fit current performance (individual needs), learners are able to work confidently at their own level of understanding while at the same time being challenged towards higher achievement.

Vygotsky's *Zone of Proximal Development* (ZPD) refers to the "distance between the actual development level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers" (Vygotsky, 1978, p. 86). Of initial importance then, is the teacher's role in determining the learner's 'zone' or level of cognitive development in the form of a diagnostic test, a class test, or based on previous results. Once the learner's level has been established, the teacher positions tasks within the ZPD – allowing access to new ideas and concepts beyond what they know, but still within reach. This is seen as the "region between the child's mastery level and instructional level, the former being that at which skills can be exercised independently, and the latter at which skills can be applied reliably only with the assistance of more capable others" (Good & Marshall, 1984, p. 12). By providing tasks that are within learners' ZPD, the teacher ensures that they make a meaningful contribution and acquire knowledge towards higher cognitive function.

Taken a step further, by identifying individual ZPD's and grouping learners according to similar levels of development, "through the social mediation of the groups interrelated ideas, the group ZPD can grow exponentially" (Schmitt, 2009, p. 37). This places learners within a 'zone' where knowledge is achievable and also allows the shift toward higher cognitive function.

3.4 Basil Bernstein

"How a society selects, classifies, distributes, transmits and evaluates the educational knowledge it considers to be public, reflects both the distribution of power and the principles of social control. From this point of view, differences within, and change in, the organization, transmission and evaluation of educational knowledge should be a major area of sociological interest" (Bernstein, 2003, p. 30).

Specialised knowledge refers to a school-code or school way of thinking, which is removed from commonsense, everyday knowledge. Acquired through a formal pedagogic relation, it is the means of transmission that leads to differentiated access to such 'specialised knowledge'. "Specialisation then reveals differences from, rather than commonality. It means that your educational identity and specific skills are clearly marked and bounded (Bernstein,1975, cited in Hoadley, 2006, p. 2).

In better understanding the acquisition of specialised knowledge, one can make use of Bernstein's theory on pedagogic discourse to analyse the transmission and acquisition of differentiated knowledge. The pedagogic discourse is defined as "an ensemble of rules or procedures for the production and circulation of knowledge within pedagogic interactions" (Singh, 2002, p. 5). This is seen as a 'recontextualisation' of knowledge through what Bernstein refers to as the *instructional* and the *regulative discourse*.

The *instructional discourse* refers to the transmission and acquisition of specific competences and skills (content knowledge) through teaching. Including aspects of selection, sequence, pacing, evaluative criteria and hierarchical rules, this then relates to the *what* and the *how* of knowledge transmitted.

The *regulative discourse* refers to the social norms and moral order of the pedagogic context. This includes expectations of manner and conduct in regulating *how* knowledge is transmitted. Here exist the hierarchical rules (the relationship between teacher and student), establishing degrees of control in the social relation. Such is the ambiguity of the regulative discourse that it is often referred to as the hidden curriculum.

$$\text{framing} = \frac{\text{instructional discourse}}{\text{regulative discourse}} \quad \frac{ID}{RD}$$

(Bernstein cited in Hoadley, 2006)

"The nature of the pedagogic discourse is that the RD always dominates the ID" (Hoadley, 2006).

This is, in some ways, linked to Vygotsky's visible and invisible mediation where there are degrees of transparency in the transmission and acquisition of knowledge.

As a means of analysis, Bernstein further developed the concepts of *classification* and *framing* of educational (specialised) knowledge.

- Classification in the broadest or most abstract of terms, 'refers to the social division of labour' (Hoadley, 2006, p. 17), bringing with it implications of power as to who gains access to what and to what extent. On a micro-level, classification refers to the degree of insulation between knowledge domains (subjects and contents). It is not 'what' is classified that is important but rather the relation between subjects or contents, and the extent of insulation between these boundaries. "Classification thus refers to the degree of boundary maintenance between contents" (Bernstein, 1975, p. 80). Within classification, boundary strength can be 'strong' or 'weak' in its levels of insulation. "Where classification is strong, contents are well insulated from each other by strong boundaries. Where classification is weak, there is reduced insulation between contents, for the boundaries between contents are weak or blurred" (Bernstein, 1975, p. 80).

- Framing refers to the strength of the boundary of knowledge transmitted and received in pedagogy. As in any educational environment, there exists a relationship between teacher and taught, which can be viewed in terms of the strength of the boundary and the "degree of control teacher and pupil possess over the selection, sequencing and pacing of the knowledge transmitted and received in the pedagogical relationship" (Bernstein, 1975, p. 80). Framing can also be 'weak' or 'strong' where 'stronger values characterise theories of instruction more centred on the transmitter, and weaker values those more centred on the acquirer' (Morais, Neves & Pires, 2004, p. 77).

Bernstein also makes the distinction between *internal* and *external* framing in decisions pertaining to the selection, sequence, pace, evaluative criteria or hierarchical rules in the pedagogic structure. External framing refers to the relations or degree of control between the teacher and the school, state or the curriculum. Internal framing relates to the relationship or degree of control between the teacher and the learner within the classroom. The analysis which follows takes into account influences of both.

With pedagogic practice described as a cultural relay, Bernstein (1990) makes the distinction between *what* (content) is relayed and *how* it (the content) is relayed. Within any pedagogic practice there exists a pedagogic relationship between the transmitter and the acquirer. Bernstein describes this relationship as 'asymmetrical' or unequal in that the transmitter can at times be seen as the acquirer and the acquirer can at times be seen as the transmitter. What is of interest is the ways in which this 'asymmetry' is masked, disguised or hidden. Bernstein expresses this cultural relay through a set of rules – hierarchical rules, sequencing rules and criterial rules - and it is the relationship within and between these rules that the pedagogic practice is defined.

As with any relationship, one has to learn how to play one's role. Bernstein (1990) argues that the transmitter needs to learn how to be a transmitter and the acquirer needs to learn how to be an acquirer. For the transmitter, one needs to learn the rules or social conditions necessary for the appropriate conduct within the pedagogic relation. These are referred to as the hierarchical rules. How the transmitter, (ie the teacher) transmits these rules and the degree of negotiation he/she may allow with the acquirer, form the regulative discourse within the pedagogic relation.

In the pedagogic relation, transmission cannot happen all at once, but requires a before and after - a progression from one 'task' to the next. Progression encompasses sequencing rules and if there is progression there is also pace. This refers to the rate of acquisition, or the amount of time available for something to be learned.

Criterial rules refer to the criteria the acquirer is expected to obtain. These describe the competence of the acquirer and his/her ability to take ownership of knowledge. These criterial rules consider the extent to which the transmitter makes explicit the criteria of a particular task and whether the acquirer has 'acquired' it. Where explicit, the learner has a clear indication as to the requirements in meeting the criteria. Where implicit, the learner is not aware of the criteria to be met.

Bernstein (1990) goes a step further in defining two types of pedagogic practice: visible and invisible. Visible pedagogic practice takes place when the rules of the instructional and regulative discourse are clear. The power relations between the transmitter and the acquirer are clear and emphasis lies on the performance of the learner and the extent to which criteria are met. Evaluative criteria are explicit where the acquirer has a clear indication of the requirements and expectations in the pedagogic relation. "He/she is not measured against himself/herself, but only against those sharing a similar temporal category" (Bernstein, 1990, p. 71).
Invisible pedagogic practice occurs when the rules of instruction are known only by the transmitter. The focus does not lie in the 'gradeable' performance of the acquirer but rather on cognitive ability and competence.

Drawing specifically on Bernstein's notion of framing, my intention is to reveal instances of power relations or degrees of control between teacher and learner through differentiated instruction of mathematics. Drawing on Bernstein's theory, Hasan argues that the "translation of power and control into principles of communication" (Hasan, 2004, p. 134) provide access to specialised knowledge at all levels of development.

3.5 Conclusion

Weaving together features of Vygotsky's *socio-cultural theory of semiotic mediation* (1978) with Bernstein's *pedagogic discourse* (1990), the intention is to decode instances of pedagogic practice where differentiation reveals itself in various forms. By constructing a model for measurement, I investigate how mediation is affected by control relations in differentiated teaching and learning and how it functions in the development of mathematical understanding. The framework measures two features. Firstly, it considers the grouping of individuals into similar ZPD's in meeting learners at appropriate levels of cognitive demand. Grouping is also analysed in terms of organisational or instructional form within the mathematics lesson. Secondly, the framework measures control relations between the teacher and learners in the pedagogic structure. By considering how knowledge is mediated through these control relations, I look at a fine-grained level of variation in mediation practices within a differentiating pedagogy.

Chapter 4: Research design and methodology

4.1 Introduction

In this chapter, I describe the method of data collection and the frameworks with which the data is analysed. The first part of this chapter describes the data sample. This specifies the school, teachers and learners involved in the case study as well as the reasoning behind its selection. It also explains the sources of the data collected and the purpose behind each of the activities. The second part of this chapter describes the *NumberSense Mathematics Programme* which serves as the model for differentiated teaching and learning in this study. It also indicates the research upon which much of this programme is based. The third part of the chapter presents the frameworks for analysis. These include the *diagnostic assessment* (Brombacher, 2018) and *instructional form* (Pedro, 1981, cited in Hoadley, 2005) which first consider the developmental and organisational aspects of grouping within a differentiated context. This follows with an integrated analysis of control relations between teacher and learners in the *framing of the pedagogic structure* (Hoadley, 2005). Together these frameworks draw points of interest from the data sample and provide a language for describing mediation.

4.2 Case study and data collection

My research takes the form of a qualitative case-study as an exploratory inquiry into instances of differentiated instruction. By collecting and analysing empirical evidence, the case-study allows for “an investigation into characteristics of real-life events where the intention is not to generalize but merely provide an account which may lead towards further inquiry” (Yin, 1984, p. 3) .

The sample of data is taken from a single school in the Western Cape, with learners of the same socio-economic status. The focus is on teaching and learning in Grade 3 mathematics in three classes with between 25 and 27 learners in each class. Three teachers took part in the case-study and are, for the purposes of this study, named Teacher A, Teacher B and Teacher C. By selecting a single school, the intention is to minimise variation at the school level and conduct an in-depth investigation of differentiating practices within a grade within a single school. In this way, the research is able to focus on differences between teachers in the practices of teaching and learning mathematics, holding dynamics of race, gender, socio-economic status, religion and cultural beliefs constant. Also, by focusing on only a single grade, the assumption is that the teachers follow the same mathematics content or curriculum within the same time frame. Of interest to this study then, are the pedagogic relations in the transmission and acquisition of mathematical knowledge and the approaches to differentiated instruction among the three teachers. The school participates weekly in the *NumberSense Mathematics Programme* whereby the teacher and learners, with support of a coach, engage in differentiated mathematics instruction. Whether or not the teachers incorporate this into their day to day mathematics lesson is unknown.

The following data was collected from each of the three classes:

- Diagnostic assessment
- Lesson observation (video-recorded)
- Teacher interview

The data collection commenced with a diagnostic assessment as a means of comparison in terms of the grouping of learners within each of the classes. The grouping prescribed by each of the teachers was compared with the results of a diagnostic assessment of the Grade 3 learners which I conducted myself. The learners were then grouped according to three or four levels of performance in mathematics and this grouping was compared with that of each of the teachers.

Further exploration looked into the different types of grouping taking place and the kinds of tasks or activities completed by each group. This grouping was measured according to an adaptation of Pedro's conceptualisation of *instructional form* (Pedro, 1981).

The main focus of the data collection consisted of the video-recorded lesson observations. A single lesson of each of the three Grade 3 teachers was video-recorded and transcribed. The mathematics lesson chosen was left to the discretion of the teacher and was approximately one hour in length. Through an adaptation of Hoadley's coding scheme, each of the lessons was analysed in terms of control relations within the pedagogic structure.

The final stage of data collection included an interview conducted with each of the Grade 3 teachers. The interview was informal and served as a means to discuss interpretations and applications of differentiated teaching and learning, as well as individual experiences in implementing such a pedagogy. The teachers were asked for their impressions about the implications for classroom management and planning, as well as future suggestions towards teaching and learning of mathematics. Certain responses to these questions are included in the data analysis in Chapter 5.

Because the research was conducted within the context of a specific mathematics intervention, I have provided a detailed description of the *NumberSense Mathematics Programme* below. Having provided support to this particular school for four years (2016 – 2019), my intention is to show *how* the programme is being implemented and to consider processes of mediation and differentiation at a more delicate and theoretical level.

4.3 The NumberSense Mathematics Programme

The NumberSense Mathematics Programme aims to be "responsive to the developmental needs of children and is informed by current research on how children learn mathematics" (Brombacher, 2011). Its focus lies in differentiated teaching and learning where mathematics is approached as "a sense-making, problem-solving activity" (Brombacher, 2011). The programme is designed with the intention that learners are grouped developmentally within the same class and work differentially, according to these levels. The NumberSense workbooks intend to match these levels and "each workbook builds on the concepts and skills developed in the previous workbook" (Brombacher, 2011).

Assuming differentiated support, the NumberSense workbooks are also designed to work in conjunction with a daily mat work routine. The intention is that the teacher works in turn with each of these developmental groups on the mat, while the other learners are engaged in independent seat work. These groups then rotate within the mathematics lesson, each working at a different pace and at different levels of cognitive demand. Below is an example of a typical NumberSense classroom:

Table 4.1: Sample of NumberSense daily mathematics routine

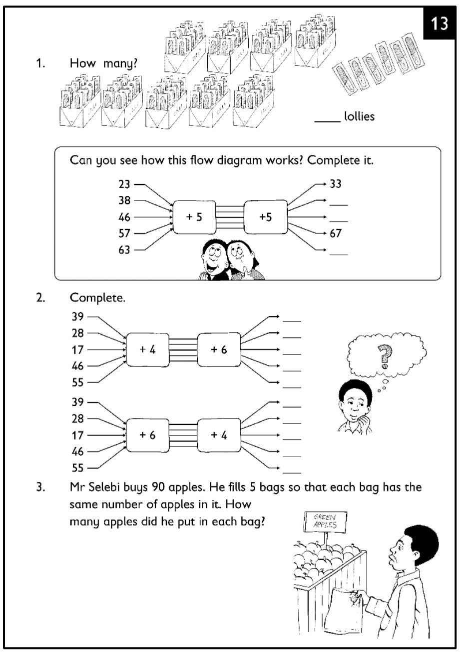
	Session 1	Session 2	Session 3
Group 1	Mat work (based on pg. 36)	Correct pg. 35 Complete pg. 36	Activity (Attribute blocks)
Group 2	Activity (Attribute blocks)	Mat work (based on pg. 13)	Correct pg. 12 Complete pg. 13
Group3	Correct pg. 2 Complete pg. 3	Activity (Attribute blocks)	Mat work (based on pg. 4)

The focus of the mat routines are specified by the programme. Learners participate specifically in three tasks: counting, manipulating number (mental mathematics) and problem solving. These tasks on the mat are aligned with the activities to be completed by learners through independent seat work in their NumberSense workbooks. Each page of these workbooks also consists of a counting, a manipulating number and a problem-solving task.

An example:
Assuming that the focus group is to complete the page shown below in Figure 4.1 (Workbook 9 Page 13), the teacher is encouraged to provide the related activites on the mat also shown in the figure. These activites are deliberately linked to the content on the given page.

Figure 4.1: Sample of NumberSense workbook page

NumberSense page



Task type	Intended mat work activity
Counting	A task involving counting in 10s E.g. piles of beans
Manipulating number	Mental calculations adding 4 and 6, and 6 and 4 and questioning whether anything is apparent.
Problem solving	Solving a similar problem also involving grouping.

The programme's intention is that the mat work prepares the learner to work independently and at a level developmentally appropriate when returning to complete the page at their desk. Having solved, shared and discussed a range of strategies on the mat, the idea is that learners are then equipped to complete the written tasks in their workbooks (Brombacher, 2011).

The NumberSense Mathematics Programme emphasises the role of routine in structuring mathematical learning. In establishing these routines, the programme provides in-service training/ coaching to a number of schools. While aligned specifically with the NumberSense workbooks, these routines are also "consistent with the expectations for early-grade mathematics teaching as described by the CAPS curriculum document" (Brombacher, 2011). Coaches provide weekly support, working alongside the teacher in his/her class, gearing them towards differentiated teaching and learning. The class is divided into groups of 8 – 10 learners, each determined by the learner's stage of development in mathematics. With the focus being the group on the mat, the coach facilitates the teacher while the other two groups are engaged in independent seat work. The intention is that these groups then rotate in such a way that each group meets with the teacher three or four times per week on the mat.

Much of the reasoning or philosophy behind the NumberSense Programme is taken from *Adding it Up* (Kilpatrick, Swaffrd & Findell, 2001) which identifies five components (strands) that work towards proficiency in mathematics. Through conceptual understanding, procedural fluency, strategic competence, adaptive reasoning and productive disposition, "the five strands provide a framework for discussing the knowledge, skills, abilities, and beliefs that constitute mathematical proficiency" (Ibid., p. 116). The programme's uptake of this framework is evident in the emphasis given to learners' ability to understand, apply and reason their way through problems. "How learners represent and connect pieces of knowledge is a key factor in whether they will understand it deeply and can use it in problem solving" (Ibid., p. 117). Evidence of these strands is visible in the data sample which follows and referred to in certain instances within the analysis.

4.4 Frameworks for analysis

The following frameworks were used in the analysis of the case study data and are discussed in greater detail below:

- Diagnostic assessment (Brombacher, 2018)
- Instructional form (Pedro, 1981, cited in Hoadley, 2005)
- Coding scheme for the framing of pedagogic practice (Hoadley, 2005)

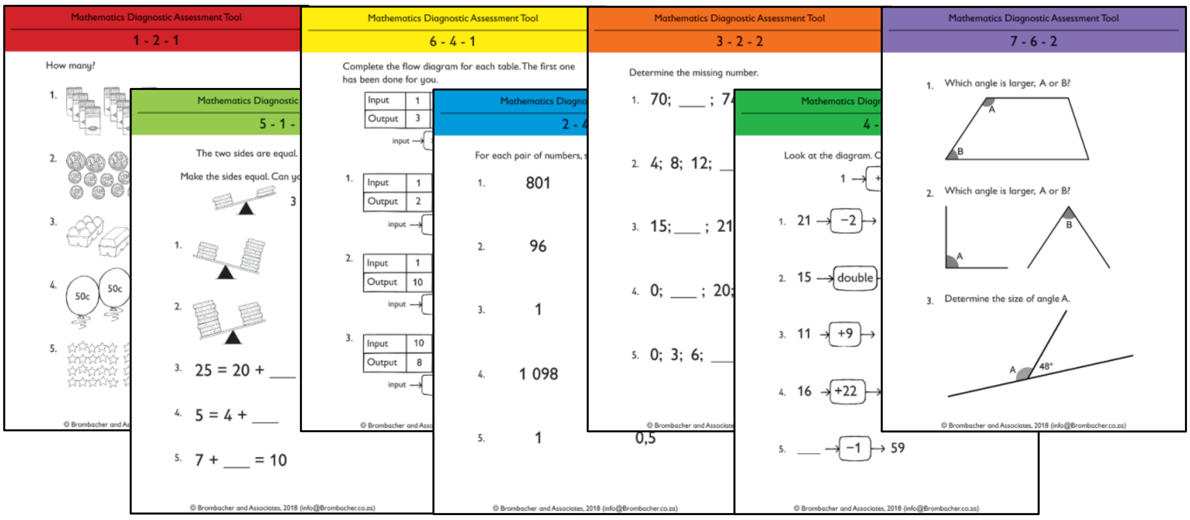
These frameworks were selected to measure various features pertaining to differentiated teaching and learning. With the intention of meeting learners at developmentally appropriate levels, the structure of the mathematics class alters, and it is through these frameworks that interpretations of differentiated instruction are measured. The *diagnostic assessment* and *instructional form* act as tools through which grouping is measured, in terms of development levels and classroom organisation. The *coding scheme of the pedagogic structure* serves to measure the control relations between the teacher and learners in the data sample. What follows is a closer account of each of these frameworks.

4.4.1 Diagnostic assessment

The first point of data collection draws on a diagnostic assessment implemented with the learners from each of the Grade 3 classes. This was used as a comparison to the grouping made by each of the teachers. The diagnostic assessment has been developed by the NumberSense Mathematics Programme to assist teachers and schools when placing learners in developmentally appropriate groups. This assessment also identifies specific skills where learners may need added support. The assessment measures performance on seven core skills predictive of future success in mathematics: counting; number comparison; number patterns; finding a rule; solving equations; formulae and geometry (Brombacher, 2018). Learners complete an assigned set of questions for each skill and are then further directed based on their responses. Learners are either guided to easier or harder sets of questions until they reach a level in which they are competent. Having completed the assessment, learners are then grouped according to their levels of development.

While this diagnostic assessment was completed digitally on a tablet by each of the learners, the example in Figure 4.2 illustrates hard copy samples of the types of questions asked in each skill.

Figure 4.2: Sample pages of each skill set



Beginning at a grade appropriate level, each learner works through the six skill sets and is digitally guided according to performance. Scoring two or less, the learner moves back towards an easier set of questions within the same skill set. Scoring four or more, the learner moves forward to a more challenging set of questions in the same skill set. Scoring three indicates that the learner is at a level where they are competent, and they proceed to the start of the next skill set. This process continues through each set in the same way. A sample of the learner's scoring path is indicated in Figure 4.3 below.

Figure 4.3: Diagnostic recording sheet

Skill Set 1: Counting

	1-R-2	1-1-1	1-1-2	1-2-1	1-2-2	1-3-1	1-3-2
1	(10) 0 1	(3) 0 1	(15) 0 1	(125) 0 1	(88) 0 1	(123) 0 1	(180) 0 1
2	(8) 0 1	(24) 0 1	(72) 0 1	(175) 0 1	(40) 0 1	(245) 0 1	(430) 0 1
3	(4) 0 1	(4) 0 1	(24) 0 1	(30) 0 1	(335) 0 1	(125) 0 1	(R6,75) 0 1
4	(24) 0 1	(25) 0 1	(100) 0 1	(130) 0 1	(105) 0 1	(285) 0 1	(570) 0 1
5	(25) 0 1	(12) 0 1	(72) 0 1	(55) 0 1	(30) 0 1	(190) 0 1	(R2,50) 0 1
	4; 5 0; 1; 2; 3	0; 1; 2 4; 5 3	0; 1; 2 4; 5 3	0; 1; 2 4; 5 3	0; 1; 2 4; 5 3	0; 1; 2 4; 5 3	0; 1; 2 3 4; 5

Skill Set 2: Comparing numbers

	2-R-1	2-R-2	2-1-1	2-1-2	2-2-1	2-2-2	2-3-1	2-3-2	2-4-1
1	(3) 0 1	(3) 0 1	(72) 0 1	(23) 0 1	(107) 0 1	(99) 0 1	(25) 0 1	(53) 0 1	(801) 0 1
2	(4) 0 1	(6) 0 1	(58) 0 1	(75) 0 1	(50) 0 1	(63) 0 1	(109) 0 1	(100) 0 1	(96) 0 1
3	(6) 0 1	(12) 0 1	(68) 0 1	(63) 0 1	(71) 0 1	(200) 0 1	(75) 0 1	(One half) 0 1	(1) 0 1
4	(5) 0 1	(5) 0 1	(11) 0 1	(100) 0 1	(58) 0 1	(120) 0 1	(150) 0 1	(354) 0 1	(2 908) 0 1
5	(11) 0 1	(41) 0 1	(92) 0 1	(110) 0 1	(91) 0 1	(213) 0 1	(200) 0 1	(201) 0 1	(1) 0 1
	4; 5 0; 1; 2; 3	0; 1; 2 4; 5 3	0; 1; 2 4; 5 3	0; 1; 2 4; 5 3	0; 1; 2 4; 5 3	0; 1; 2 4; 5 3	0; 1; 2 4; 5 3	0; 1; 2 4; 5 3	0; 1; 2 3; 5

Having completed the test, a modal score is indicated which suggests the learner's approximate level of mathematical development. Learners are then grouped according to three groups of achievement – high, average and low achievement. While used as a basis for grouping learners in this study, it also serves a tool which indicates specific skills or areas of mathematics where individuals may need attention. These skills can then be attended to in smaller groups with the teacher.

By placing learners in groups of similar developmental levels, the intention is that the teaching and learning of each group takes place at a level appropriate to understanding. This links to Vygotsky's *Zone of Proximal Development* in meeting learners at a developmentally appropriate level and guiding them towards higher levels of cognitive development. "For a teacher to teach to a student's zone of proximal development, first the teacher must determine what that zone is" (Lin & Small, 2010, p. 3). While assigned to particular groups, the intention is that these groups also remain fluid, where learners are able to move up or down depending on their developmental needs. In the analysis in Chapter 5, the results of the diagnostic assessment are compared with the teachers' grouping of the learners. Similarities and differences are discussed as well as possible reasoning behind instances of disparity.

(In developing the diagnostic tool, reference is made to Spearman's rank correlation as well as research investigating factors crucial to the development of foundational mathematical skills (Aunio & Räsänen, 2015). This diagnostic assessment is currently in its pilot stage while being trialed at a number of schools and will be further developed upon verification on a broader scale. It was however deemed sufficiently valid for use in this research).

4.4.2 Instructional form

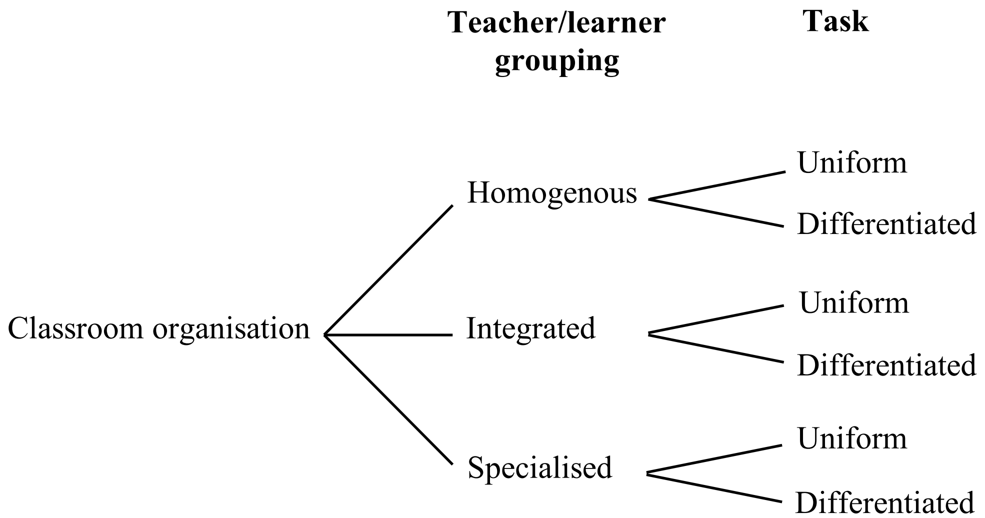
The second point of analysis draws on Pedro's (1981) conceptualising of *instructional form*. This incorporates the organisational aspect of teaching where one "extends the discussion of the classification of agents, or the extent to which the pedagogic identities of students are demarcated" (Hoadley, 2005, p. 137). In her study of *Social stratification and classroom discourse*, Pedro (1981) aims to show the differences between classrooms. Based on Bernstein's instructional and regulative discourse, Pedro makes parallels with instructional and moral order and how "social classes are reproduced by the education process" (Pedro, 1981, p.

291). Pedro talks of two forms of differentiation, the outer form concerning the frames of the state, the curriculum or the timetable; and the inner form which relates to the social positioning of learners.

Pedro also incorporates organisational (instructional) form which refers to the way in which learners are grouped in a classroom for pedagogic purposes. Pedro makes distinctions between *homogenous*, *integrated* and *specialised* teaching. Homogenous refers to whole-class teaching, integrated refers to groups working together and specialised refers to the teacher working specifically with groups or individuals. Pedro then considers the differentiation of content in terms of different tasks / activities assigned to different learners or groups. This is classified as uniform or differentiated.

Figure 4.4: Scheme for the analysis of instructional form

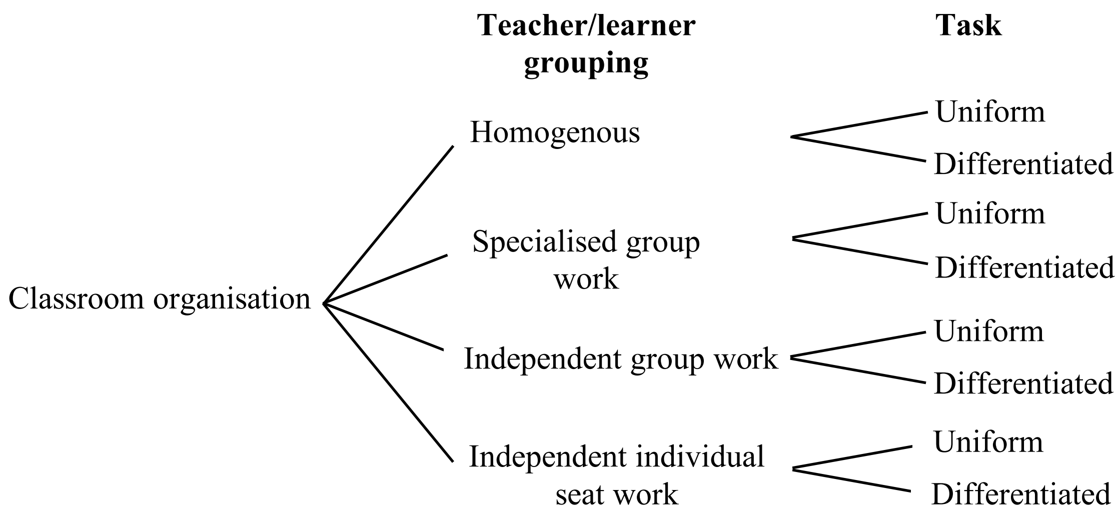
(Pedro, 1981, cited in Hoadley, 2005)



For this study, my aim is to consider how learners are grouped within a mathematics lesson, not according to social positioning but rather in terms of developmental levels, and how content is distributed between these groups. For the purposes of this study, Pedro's scheme for analysis of instructional form has been adapted to suit the types of grouping that takes place in each of the selected Grade 3 classroom settings as illustrated below.

Figure 4.5: Adaptation of Pedro's scheme for analysis of instructional form

(Pedro, 1981, cited in Hoadley, 2005)



As per Pedro's model, *homogenous* refers to a whole-class teaching approach. *Specialised group work* refers to the teacher working with a group of learners on the mat and is regarded as the focus group for that section of the lesson. *Independent group work* involves a group working together on the same activity but without the assistance of the teacher. Finally, *independent individual seatwork* involves learners working on their own in their workbooks on a particular page prescribed by the teacher at the beginning of the lesson.

Through careful consideration of the grouping of learners, both developmentally and organisationally, the intention is to analyse the instructional form upon which differentiated teaching and learning is built.

4.4.3 Coding scheme for analysis of the pedagogic structure

The third point of analysis provides a Bernsteinian inquiry into the framing of the pedagogic structure using an adaptation of Hoadley's coding scheme (2005). The focus lies only on the specialised group activities taking place with the teacher on the mat and the extent to which control relations vary between groups and/or tasks.

Drawing specifically on five aspects of framing in the pedagogic structure, the framework aims to illustrate and compare the three teachers' interpretations of differentiated instruction. These aspects are the selection, sequence, pacing, evaluative criteria and the hierarchical rules in the pedagogic structure as represented in Table 4.2 below.

Table 4.2: Adaptation of the coding scheme for framing (Hoadley, 2005, p. 311)

Framing of the pedagogic structure
The extent to which the teacher controls the **selection** of content
The extent to which the teacher controls the **sequence** of content
The extent to which the teacher controls the **pacing** of content
The extent to which the teacher makes the **evaluative criteria** explicit to learners
The extent to which the teacher controls the **hierarchical rules** between teacher and learner

In order to measure the pedagogic structure of each of the teachers systematically, the three lessons were transcribed and categorised according to the groups within each class. The data within each of these groups was then organised into tasks which formed the unit of analysis. These tasks were defined in terms of counting, manipulating number and problem-solving tasks.

Each of the tasks was coded according to 11 indicators. Each of these indicators are scalar and have four degrees of control (from very weak to very strong). Adapting Hoadley's (2005) coding scheme for Bernstein's concepts, the intention is to provide a grammar for analysis of the control relations between teacher and learners and the implications for differentiation in mathematics. The framing scales measure the strength or weakness in the degree of control between teacher and learners. Where framing is coded F^{++}, this indicates very strong teacher control, while F^{--} indicates very weak teacher control. The complete coding scheme, illustrating each of the five aspects and their corresponding indicators, can be found in the Appendix. The example in Table 4.3 below illustrates one of the four indicators in the evaluative criteria.

Table 4.3: An example of the development of a coding scheme

Discursive rule **EVALUATIVE RULES** (F^{+-})
The extent to which teacher and learner have control over the evaluative criteria of the instructional knowledge pertaining to the meaning of concepts and principles and their appropriate realization

	F^{++}	F^{+}	F^{-}	F^{--}
4. In the introduction/ explanation/ exposition to a topic/task	**Evaluative criteria very clear and explicit**	**Evaluative criteria quite clear and explicit**	**Evaluative criteria quite unclear and implicit**	**Evaluative criteria very unclear and implicit**
	The teacher always or almost always makes the evaluative rules available through exposition. The teacher explicitly defines and explains the meaning of concepts, addressing key aspects of the knowledge or operation under discussion. She makes it clear exactly how a task should be completed.	Most of the time the teacher makes the evaluative rules available in an explicit and clear manner. The requirements for the successful completion of a task are generally clear, although there may be some aspects that remain implicit.	The concepts and principles being addressed in the exposition are sometimes unclear. Attempts are made to make the requirements for the successful production of a text available to learners, but these are often unclear or not articulated. Some ambiguity as to what should be done and how it should be done exists.	Generally, the teacher does not draw out the knowledge principles in her exposition. Very little or no attempt is made to make the requirements for the successful production of a text available to learners. Learners are unclear as to how to proceed or continue in any manner they choose.

Table 4.4 below provides a sample of three tasks coded according to control relations in the evaluative criteria and how it is read in tabulated form. The evaluative rules consist of four indicators (labelled 1, 2, 3, 4 in the table). A framing code is assigned for each indicator in the rows below, for the three groups indicated in the left-hand column. This example considers control relations of just one of the teachers. It indicates that Teacher A worked with three groups in the lesson – Group 1, Group 2 and Group 3. Each of these groups was measured according to counting, manipulating number and problem-solving tasks. The example illustrates that Group 1 did not partake in any counting tasks but completed one manipulating number task and two problem-solving tasks. Group 2 and Group 3 each completed one counting, one manipulating number and one problem solving task. The codes (F^{+-}) measure the strength of control relations between teacher and learners within each of these tasks. The final row highlighted in blue indicates possible trends across the three groups based on an aggregation of the framing codes assigned.

Table 4.4: An example of the coding scheme for evaluative criteria

<table>
<tr><th></th><th></th><th colspan="4">Counting</th><th colspan="4">Manipulating number</th><th colspan="4">Problem solving</th></tr>
<tr><th></th><th>Indicator</th><th>1</th><th>2</th><th>3</th><th>4</th><th>1</th><th>2</th><th>3</th><th>4</th><th>1</th><th>2</th><th>3</th><th>4</th></tr>
<tr><td rowspan="5">Teacher A</td><td rowspan="2">Group 1</td><td colspan="4" rowspan="2">-</td><td rowspan="2">F^{+}</td><td rowspan="2">F^{-}</td><td rowspan="2">F^{--}</td><td rowspan="2">F^{+}</td><td>F^{--}</td><td>F^{++}</td><td>F^{++}</td><td>F^{+}</td></tr>
<tr><td>F^{-}</td><td>F^{++}</td><td>F^{++}</td><td>F^{+}</td></tr>
<tr><td>Group 2</td><td>F^{+}</td><td>F^{--}</td><td>F^{+}</td><td>F^{+}</td><td>F^{+}</td><td>F^{-}</td><td>F^{+}</td><td>F^{+}</td><td>F^{-}</td><td>F^{+}</td><td>F^{++}</td><td>F^{+}</td></tr>
<tr><td>Group 3</td><td>F^{+}</td><td>F^{--}</td><td>F^{+}</td><td>F^{+}</td><td>F^{-}</td><td>F^{-}</td><td>F^{+}</td><td>F^{-}</td><td>F--</td><td>F^{++}</td><td>F^{++}</td><td>F^{++}</td></tr>
<tr><td>Trend</td><td>F^{+}</td><td>F^{--}</td><td>F^{+}</td><td>F^{+}</td><td>F^{+}</td><td>F^{-}</td><td>F^{+}</td><td>F^{+}</td><td>$F^{-/--}$</td><td>F^{++}</td><td>F^{++}</td><td>F^{+}</td></tr>
</table>

By measuring each of the specialised group tasks according to the coding scheme and comparing control relations between teacher and learners, this study looks at fine-grained variation in mediational practices within a differentiating pedagogy.

4.5 Conclusion

In this chapter, I have sought to position the data for analysis in the context of differentiated instruction. By describing the data sample, the NumberSense Mathematics Programme and the frameworks through which the data is analysed, I have tried to situate the reader within the context for the analysis. Using these frameworks in the analysis that follows, I first consider the grouping of learners, in terms of levels of cognitive development and in terms of classroom organisation. Secondly, I consider the control relations between the teacher and learners within these groups and according to particular tasks. By considering each of these elements in the analysis chapter that follows, I consider how productive the frameworks are for investigating mediational practices within a differentiating pedagogy.

Chapter 5: Data analysis

5.1 Introduction

In this chapter, I describe the pedagogic structure of the three teachers in the data sample. Through a set of frameworks for measurement, my analysis aims to define and draw instances of control relations through:

- the grouping of the learners and
- the framing of the pedagogic structure.

In terms of grouping, the data is measured according to the teachers' grouping of the learners in relation to the grouping suggested by the *diagnostic assessment.* Analysis also considers the organisational grouping within each class according to an adaptation of Pedro's *instructional form*. This describes the activities taking place simultaneously between the different groups and the organisational form of each group.

Having established the differentiated grouping of learners within each class, analysis then measures the framing of the pedagogic structure in terms of the group working with the teacher on the mat. This indicates the control relations between teacher and learners in terms of the selection, sequence, pace, evaluative criteria and hierarchical rules within particular groups or tasks. Through an adaptation of *Hoadley's coding scheme* for Bernstein's concepts (2005), the intention is to reveal forms of differentiating pedagogy in mathematics.

In summarising the data analysis, the sample suggests an 'ideal pedagogy' in differentiated teaching and learning of early-grade mathematics through shifting control relations in the pedagogic structure. This indicates a framework through which mediational practices can be measured. This is elaborated in the discussion which follows in Chapter 6.

5.2 Grouping of learners

5.2.1 Teacher grouping

Each of the three classes was divided into three groups during the mathematics lesson. The groups were organised according to performance in mathematics - high, average and low achievement. On interviewing each of the teachers, they reported that a baseline assessment was initially used to group the learners. Movement between the groups was also said to remain fluid, where learners may be reassigned based on performance or as means of challenging some or building confidence in others. Teacher A indicated that the baseline results were also gauged against the Grade 2 Term 4 assessments before grouping her learners. Teacher B allocated groups according to what she called a "gut feel" and Teacher C made later use of the Term 1 assessments to adjust grouping more accurately.

Table 5.1: Teacher grouping of learners for mathematics lessons

	Group 1	**Group 2**	**Group 3**
Teacher A	Red	Green	Blue
Teacher B	Proteas	Roses	Daisies
Teacher C	Springboks	King Proteas	Blue cranes

5.2.2 Teacher grouping in relation to the diagnostic assessment

As a means of comparison, grouping made by the teachers was measured against the results of a diagnostic assessment developed by the NumberSense Mathematics Programme. As each of the teachers had grouped learners according to three levels of development, so too were the results from the diagnostic divided into three levels of achievement. The results of these comparisons are as follows:

Table 5.2: Teacher A's grouping of learners in relation to diagnostic assessment

Class A		
Learners matching diagnostic level	**Learners grouped below diagnostic level**	**Learners grouped above diagnostic level**
14 of 25	4 of 25	7 of 25
56%	16%	28%

Table 5.3: Teacher B's grouping of learners in relation to diagnostic assessment

Class B		
Learners matching diagnostic level	**Learners grouped below diagnostic level**	**Learners grouped above diagnostic level**
13 of 24	3 of 24	8 of 24
54%	13%	33%

Table 5.4: Teacher C's grouping of learners in relation to diagnostic assessment

Class C		
Learners matching diagnostic level	**Learners grouped below diagnostic level**	**Learners grouped above diagnostic level**
19 of 27	3 of 27	5 of 27
70%	11%	19%

Results from Class A illustrate that 56% (14 out of 25) of the learners were grouped in alignment with the diagnostic assessment while 16% (4) were grouped below and 28% (7) above. In the case of Class B, 54% (13 out of 24) of the learners were grouped in alignment with the diagnostic assessment, 13% were grouped below and 33% above. For Class C, 70% (19 out of 27) of learners were aligned with the diagnostic assessment, 11% of learners (3) were in grouped in a level below that of the diagnostic, while 19% (5) were grouped in a level above.

Noticing disparities, further comparison was made in the attempts to draw out the degree to which the teacher's grouping differed from that of the diagnostic assessment, ie. whether learners were grouped one or two levels above or below that of the diagnostic assessment. This sought to highlight instances where, for example, learners were in the teacher's top group, but the diagnostic assessment indicated that they had scored in the bottom group or vice versa.

Table 5.5: Teacher A's grouping of learners in relation to diagnostic assessment

Class A		
Learners matching diagnostic level	**Learners 1 level above/below diagnostic**	**Learners 2 levels above/below diagnostic**
14 of 25	11 of 25	0 of 25
56%	44%	0%

Table 5.6: Teacher B's grouping of learners in relation to diagnostic assessment

Class B		
Learners matching diagnostic level	**Learners 1 level above/below diagnostic**	**Learners 2 levels above/below diagnostic**
13 of 24	9 of 24	2 of 24
54%	38%	8%

Table 5.7: Teacher C's grouping of learners in relation to diagnostic assessment

Class C		
Learners matching diagnostic level	**Learners 1 level above/below diagnostic**	**Learners 2 levels above/below diagnostic**
19 of 27	8 of 27	0 of 27
70%	30%	0%

For Class A, while 44% (11 out of 25) of the learners were grouped one level above or below that of the diagnostic, none were grouped two levels above or below that of the diagnostic. This indicates that the degree to which grouping differed was not substantial. For example, a learner placed in the top group by the teacher may have scored as medium ability in the diagnostic test, but there are no instances where they were found in the bottom group. For Class B, 38% were grouped one level above or below that of the diagnostic test while 8% of the learners were grouped two levels above or below. For Class C, 30% (8) of the learners were grouped one level above or below that of the diagnostic but none were placed in groups two levels above or below that of the diagnostic.

In the cases where learners were placed in groups two levels higher/lower to that of the diagnostic assessment (for Class B only), reasoning was queried in the interview with the teacher. Teacher B indicated that one of the learners was frequently absent while the other had dropped dramatically in achievement since the beginning of the year due to issues at home. Based on classroom observation which follows, Teacher B displays an uneven distribution across the groups. Her second group is too large and her third group has only three learners. On interviewing Teacher A and Teacher C, absenteeism was also indicated as an influencing factor. Teacher A indicated that some of her learners had been pushed to the top group as a means of challenging and boosting learners' confidence. She also remarked that some of her learners held the potential to be in the top group but were inconsistent in their performance, so were shifted down a level.

Differences could also include technological advantages or limitations (bearing in mind that the assessment took place on a tablet), or perhaps the learner's state of mind on that particular day. One should also anticipate that by the third term, a teacher would have a better indication of her learners' developmental levels or the extent to which they can or should be challenged.

Upon comparison of the three teachers, reasoning for grouping reveals both academic and social criteria. The grouping of Teacher A and Teacher C is more closely aligned to the diagnostic assessment and is based primarily on academic criteria. The grouping of Teacher B is less aligned with the diagnostic assessment and is based on both academic and social criteria. When interviewing Teacher A and Teacher C, both reported to engage in group work on a regular basis. There is evidence of this in the classroom observations that follow where both of these teachers display evidence of a firmer hold on the dynamics of each of the groups. Teacher B reported that she does not engage in regular group work and focuses more on whole-class teaching. This perhaps makes cognisance of the developmental levels of each learner less important.

5.2.3 Instructional form

Instructional form refers to the way in which groups are situated within each class as well as the content with which they are engaged. As each of the three teachers engage in within-class grouping or differentiation, the model below aims to better define *how* these groups are organised. "The analysis of instructional form presented here thus focuses on differentiation – between agents (students) and between contents (knowledge)" within each of the three classes (Hoadley, 2005, p. 138). In characterising the instructional form, I drew on the classroom observation data of the three teachers across the three days.

In class A, the groups partake in specialised and independent group work as well as independent individual seat work, rotating in such a way that each of the groups participate in all of the tasks. The specialised group work refers to those learners working on the mat with the teacher, while the second group works independently in their NumberSense workbooks and the third group completes a group practical activity. In such a way, the first group partakes in specialised group work, the second in independent individual seat work and the third in independent group work. The groups then rotate until all the groups have completed all the activities or tasks.

Table 5.8: Instructional form of Class A

	Class A		
	Specialised group work	**Independent individual seat work**	**Independent group work**
Session 1	Blue group	Red group	Green group
Session 2	Green group	Blue group	Red group
Session 3	Red group	Green group	Blue group

In class B, the lesson begins with homogenous teaching. The teacher works with the whole class in an oral counting and mental mathematics activity. The teacher then divides the class in such a way that the first group is specialised in working on the mat, while the second and third groups are involved in independent individual seat work activities. They complete a page in their NumberSense workbooks and a page in their DBE books. The groups then rotate where half of the second group (it was mentioned that this group had become too large) works on the mat with the teacher while the other two groups complete their allocated pages. In the case of class B however, the groups only rotate twice and neither the second half of the second group nor the third group work with the teacher on the mat.

Table 5.9: Instructional form of Class B

<table>
<tr><th></th><th colspan="8">Class B</th></tr>
<tr><th></th><th>Homogenous</th><th>Specialised group work</th><th colspan="6">Independent individual seat work</th></tr>
<tr><td>Session 1</td><td>Whole class</td><td>-</td><td colspan="6">-</td></tr>
<tr><td>Session 2</td><td></td><td>Proteas</td><td colspan="3">Roses</td><td colspan="3">Daisies</td></tr>
<tr><td>Session 3</td><td></td><td>Roses (girls)</td><td colspan="2">Proteas</td><td colspan="2">Roses (boys)</td><td colspan="2">Daisies</td></tr>
</table>

Teacher C works in the same way as Teacher A where each of the groups partake in a specialised group activity, an independent group activity and an independent individual seat work activity.

Table 5.10: Instructional form of Class C

	Class C		
	Specialised group work	**Independent individual seat work**	**Independent group work**
Session 1	Springboks	King proteas	Blue cranes
Session 2	Blue cranes	Springboks	King proteas
Session 3	King proteas	Blue cranes	Springboks

As in the grouping criteria, most disparity lies with Teacher B. While the teacher has divided her class according to three ability groups, these are uneven and observed to be largely mismanaged. While the top group seems to best fit the profile and are also matched in the diagnostic test, the middle group is the largest of the groups, to the point that this group had to be further split into boys and girls to accommodate the mat routine. Only the girls took part in the mat routine on that day. The teacher seemed surprised by the size of the group and suggested that they be reassigned or shifted to another group. The weakest groups consisted of only three learners and they did not take part in the mat routine on that particular day. Upon interviewing Teacher B, it was made clear that group teaching was not a priority and that she preferred a whole class teaching approach. Teacher B reported in her interview that she believes there is merit in group work but only as an after-school intervention for those learners needing additional mathematics tuition.

In summary, grouping of learners' points to similarities between Teacher A and Teacher C. In terms of the teacher's grouping of learners, Teacher A and Teacher C display a firm grasp of the dynamics of the groups – the number of learners and levels of ability. Teacher B however, displays a lesser knowledge of her groups. In relation to the diagnostic assessment, Teacher A and Teacher C also reveal a closer match than Teacher B. This could be as a result of Teacher B's lack of management of her own grouping, or the infrequency of group work. In terms of the instructional form within each of the classes, Teacher A and Teacher C showcase three distinct groups within the class, each focusing on different activities at different points in the lesson. While Teacher B also shows evidence of three different groups, she includes homogenous teaching as part of her lesson rather than integrated group work.
In the next section, the analysis measures framing of the pedagogic structure in the specialised group work, where forms of pedagogy are revealed.

5.3 Framing of the pedagogic structure

In this section, analysis measures control relations in the selection, sequence, pace, evaluative criteria and the hierarchical rules in the pedagogic structure of the three classrooms. For the purposes of this analysis, only the specialised group working with the teacher on the mat is included. It does not take into account the tasks and activities of the remainder of the class. The aim is to consider aspects of differentiation in terms of the groups, the teachers and the types of tasks (counting, manipulating number and problem solving) in the mat work routine. Each of these aspects are explained in greater detail and in relation to particular tasks based on the findings in the data collection. (See Appendix for the detailed coding scheme).

5.3.1 Selection and sequence

Framing over the selection and sequence of instructional knowledge in the pedagogic structure each consist of a single indicator. The first indicator relates to the *selection* of tasks and the degree to which the teacher allows learners to exercise choice over the content of the lesson or task. Where framing is strong (F^{++}), the teacher holds control over the selection of tasks, activities and knowledge in the group. Learners rarely interrupt and interjections are ignored. Where framing is weak (F^{--}), learners often make decisions around the selection of tasks and the teacher alters selection accordingly. The second indicator considers the extent to which the teacher controls the *sequence* of tasks or knowledge within the group. Where framing is strong, the teacher always determines the sequence of transmission of knowledge and learner interjections are dismissed. Where framing is weak, learners have the opportunity to vary the sequence of transmission. The table below provides a summary of framing over the selection and sequence of tasks for each teacher and each group.

Table 5.11: Selection and sequence in the pedagogic structure

		Counting		**Manipulating number**		**Problem solving**	
		Selection	**Sequence**	**Selection**	**Sequence**	**Selection**	**Sequence**
Teacher A	Group 1			F^{++}	F^{+}	F^{++}	F^{++}
						F^{++}	F^{++}
	Group 2	F^{++}	F^{++}	F^{++}	F^{++}	F^{++}	F^{++}
	Group 3	F^{++}	F^{++}	F^{++}	F^{++}	F^{++}	F^{+}
	Trend	$\mathbf{F^{++}}$	$\mathbf{F^{++}}$	$\mathbf{F^{++}}$	$\mathbf{F^{++}}$	$\mathbf{F^{++}}$	$\mathbf{F^{++}}$
Teacher B	Group 1	-	-	F^{++}	F^{++}	F^{++}	F^{+}
						F^{++}	F^{++}
	Group 2	-	-	F^{++}	F^{++}	-	-
				F^{++}	F^{+}		
				F^{+}	F^{+}		
	Trend	**-**	**-**	$\mathbf{F^{++}}$	$\mathbf{F^{+/++}}$	$\mathbf{F^{++}}$	$\mathbf{F^{++}}$
Teacher C	Group 1	F^{++}	F^{+}	F^{++}	F^{++}	F^{+}	F^{+}
				F^{+}	F^{+}	F^{++}	F^{++}
	Group 2	F^{++}	F^{++}	F^{++}	F^{++}	F^{++}	F^{+}
	Group 3	F^{++}	F^{+}	F^{++}	F^{+}		
				F^{+}	F^{+}		
	Trend	$\mathbf{F^{++}}$	$\mathbf{F^{+}}$	$\mathbf{F^{++}}$	$\mathbf{F^{+}}$	$\mathbf{F^{++}}$	$\mathbf{F^{+}}$
Overall trend		$\mathbf{F^{++}}$	$\mathbf{F^{+/++}}$	$\mathbf{F^{++}}$	$\mathbf{F^{+/++}}$	$\mathbf{F^{++}}$	$\mathbf{F^{++}}$

The framing over selection and sequence holds strong across tasks, groups and teachers in the data sample. All of the teachers are explicit in both the content to be covered as well as the order in which tasks take place. With indicators F^{+} or F^{++} throughout, content and meaning is firmly selected and organised by the teacher.

One could argue however, that framing over selection is largely determined by the textbook (NumberSense workbook). The tasks which take place on the mat are, in most cases, preparation for the activities to be completed independently in learners' workbooks. If considering internal versus external framing over the selection and sequence of tasks however, there is evidence of slight variation in the strength or level of control. Externally, framing is strong in that the school/management has stipulated that the workbooks be used. Internally however, the teacher holds a greater authority or control as to the *level* of content allocated to each group. It is here where she exercises the control to differentiate teaching and learning by selecting different workbooks for different groups.

Table 5.12: Selected pages to be completed by each group

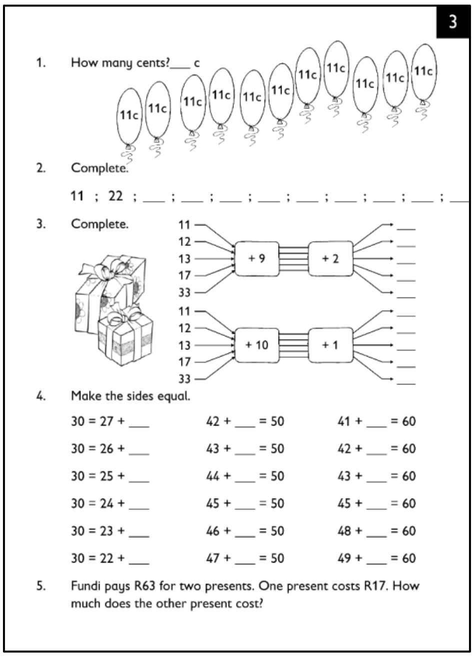
	Group 1	**Group 2**	**Group 3**
Teacher A	NS Workbook 10 Pg 14	NS Workbook 9 Pg 41	NS Workbook 9 Pg 20
Teacher B	NS Workbook 9 Pg 37	NS Workbook 9 Pg 30	NS Workbook 9 Pg 22
Teacher C	NS Workbook 10 Pg 3	NS Workbook 9 Pg 18	NS Workbook 9 Pg 11

Table 5.12 above indicates that each of the groups in the three classes work in different workbooks, or at different stages within a workbook. For example, Teacher A's top performing group is working in Workbook 10, the middle group in Workbook 9 on Page 41 while the lower achieving group are also working in Workbook 9 but on Page 20. The specialised mat routines are centred around the page allocated for each group, where the tasks which take place on the mat prepare the learners for working independently on their pages. Teacher A and Teacher C show a broader scope in levels of development between the groups, with the top group working in Workbook 10 and the middle and lower groups in Workbook 9. Teacher B shows slightly less variation in that all the groups are at different points but within the same workbook.

In some tasks, there exist slight shifts in the degree of control over selection or sequence (F^{++} to F^{+}) where the teacher may allow certain interjections or productions. She may, for example, allow the learners to select their own numbers/values within a given task, but the topic or concept for the lesson remains fixed. This is illustrated in the example below.

Figure 5.1: NumberSense page in relation to mat work activities

Allocated NumberSense page **Mat work activities**

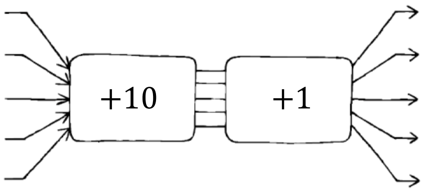


Counting — Adding 11s

Manipulating number — Multiples of 11

How many 11s in a number?

Flow diagram activity adding 11s:



Problem solving

Based on the content of Workbook 10 Page 3, Teacher A develops a counting activity where adding 11s relates to the counting in intervals of 11 cents in their NumberSense workbook. In the manipulating number activity, the teacher presents an oral activity with multiples of 11. This is also reflected on the page. The teacher then consolidates with a flow diagram activity. Providing each with a template, the teacher allows the learners to select their own input numbers when completing the activity as a means of gauging understanding.

External framing is therefore strong in that the NumberSense workbook is stipulated for use, however, teachers have some control in the selection of workbooks and types of tasks used for each of the groups. Sequencing remains for the most part unchanged in that the teacher and her groups follow the order of the tasks in the workbook when working on the mat.

5.3.2 Pace

Framing over *pacing* consists of a single indicator, considering the extent to which the teachers or learners exercise control over the rate of transmission when completing a particular task. Where framing is strong, the teacher strictly controls the pace at which the learners work and allows for little or no interruption. Reference to time is frequent and the teacher does not vary pace according to learners' productions. Framing is considered weak when learners work at their own pace and the teacher allows for interjections and discussion before moving on to the next task. The teacher places no time constraints on finishing a task within a given period of time.

The table below provides a summary of framing over pacing in the mat routines conducted by each of the teachers.

Table 5.13: Pacing in the pedagogic structure

<table>
<tr><th colspan="2"></th><th>Counting</th><th>Manipulating number</th><th>Problem solving</th></tr>
<tr><th colspan="2"></th><th colspan="3">Pacing</th></tr>
<tr><td rowspan="5">Teacher A</td><td rowspan="2">Grp 1</td><td rowspan="2">-</td><td rowspan="2">F^+</td><td>F^-</td></tr>
<tr><td>F^-</td></tr>
<tr><td>Grp 2</td><td>F^+</td><td>F^+</td><td>F^-</td></tr>
<tr><td>Grp 3</td><td>F^+</td><td>F^-</td><td>F^-</td></tr>
<tr><td>Trend</td><td>F^+</td><td>F^+</td><td>F-</td></tr>
<tr><td rowspan="6">Teacher B</td><td rowspan="2">Grp 1</td><td rowspan="2">-</td><td rowspan="2">F^+</td><td>F^-</td></tr>
<tr><td>F^+</td></tr>
<tr><td rowspan="3">Grp 2</td><td rowspan="3">-</td><td>F^+</td><td rowspan="3">-</td></tr>
<tr><td>F^+</td></tr>
<tr><td>F^+</td></tr>
<tr><td>Trend</td><td></td><td>F^+</td><td>$F^{+/-}$</td></tr>
<tr><td rowspan="6">Teacher C</td><td rowspan="2">Grp 1</td><td rowspan="2">F^+</td><td>F^+</td><td>F^-</td></tr>
<tr><td>F^-</td><td>F^-</td></tr>
<tr><td>Grp 2</td><td>F^+</td><td>F^-</td><td>F^-</td></tr>
<tr><td rowspan="2">Grp 3</td><td rowspan="2">F^+</td><td>F^+</td><td rowspan="2">-</td></tr>
<tr><td>F^-</td></tr>
<tr><td>Trend</td><td>F^+</td><td>F^-</td><td>F^-</td></tr>
<tr><td colspan="2">Overall trend</td><td>F^+</td><td>F^+</td><td>F^-</td></tr>
</table>

In counting and most manipulating number tasks, all three of the teachers demonstrate strong control over pacing, between classes and also between groups. There is little room for learners to interrupt or delay the task and a higher level of attention is required in keeping up with the questions asked and the topic to which it relates. A general pattern involves the teacher moving quickly around the group, asking closed questions and only slightly weakening control when asking children to explain their thinking.

Below are three examples of strong degrees of control (F^{+}) over pacing in manipulating number tasks across the three teachers.

Extract 5A:

Teacher A

Teacher:	I want to make 20. I give a number and you give a number. Right? 16
Learner:	4
Teacher:	18
Learner:	What teacher?
Teacher:	Hayibo! Ok, let's start again. I want 20. I give a number. You give a number. Right? Ok. 16
Learner:	4
Teacher:	15
Learner:	5
Teacher:	3
Learner:	17
Teacher:	19
Learner:	1
Teacher:	8
Learner:	12
Teacher:	6
Learner:	14
Teacher:	9
Learner:	11
Teacher:	11
Learner:	9
Teacher:	Well done. 13
Learner:	7
Teacher:	Grade 3's well done! Good work. You can go back to your seat.

Teacher B

Teacher:	Tell me if I double 8 what do I get?
Learner:	16
Teacher:	Good. If I double 7 what do I get?
Learner:	14
Teacher:	If I double 20 what do I get?
Learner:	40
Teacher:	If I halve 60 what do I get?
Learner:	25
Teacher:	Think again. Half of 60. Half of 60?
Learner:	30
Teacher:	30. What is half of 50?
Learner:	25
Teacher:	25. What is half of 70?
Learner:	*(Learner mumbles)*
Teacher:	I didn't hear that?
Learner:	35
Teacher:	35. Um… What is 12 doubled?
Learner:	24
Teacher:	24. What is 15 doubled?
Learner:	30
Teacher:	Abigail try again… What is 21 doubled?
Learner:	42

Teacher C

Teacher:	What is 6 times 11?
Learner:	66
Teacher:	5 times 11?
Learner:	55
Teacher:	3 times 11?
Learner:	33
Teacher:	7 times 11?
Learner:	77
Teacher:	6 times 11?
Learner:	66
Teacher:	8 times 11?
Learner:	88
Teacher:	9 times 11?
Learner:	99
Teacher:	And 10 times 11?
Learner:	110.
Teacher:	Ok what must 2 be multiplied by to get 22?
Learner:	11
Teacher:	11. What must 3 be multiplied by to get 33?
Learner:	11.
Teacher:	What must 6 be multiplied by to get 66?
Learner:	11

Each of the teachers work closely to the expectations of the NumberSense Mathematics Programme, not only in terms of pace, but also according to the content suggested for mental mathematics development. Teacher A presents a task with bonds of 20, Teacher B focuses on doubling and halving, while Teacher C works with multiples of 11. Control over pacing is strong with the expectation of developing procedural fluency and being able to apply these skills later in problem-solving tasks.

In contrast, control over pacing tends to weaken during problem-solving tasks. While this is evident with all teachers, there is slight variation in the strength of control with Teacher A and Teacher C in comparison to that of Teacher B. The following example highlights weak framing over pacing in a particular problem-solving task.

Teacher A presents the group with a problem and asks the learners to 'make a plan'.

Extract 5B:

> Teacher: Amila wants to buy herself a new t-shirt. The t-shirt costs R100. Amila saved some of her money. She already saved R45. How much money must Amila still save to buy the t-shirt? The t-shirt is R100, she already saved R45. How much money must she still save? Can you quickly show me? Make a plan there.

Teacher A then walks around the group, observing learners as they work out the problem in their mat books and looks for particular strategies upon which to draw. She repeats the problem and continues to look for a range of appropriate solutions.

> Teacher: Amila wants to buy a t-shirt. The t-shirt cost R100. How much money must Amila still save to buy her t-shirt. Can anyone give me an answer? Let me see.
> Ah!! I like your plan. Go on. Go on. Well done. Quick quick!
>
> *Teacher walks around to check on class*
>
> That's right Amila. Quick my child. Ok. Let me see what are you doing? I can't read. You must write a bit bigger but I like this plan also. I just can't read everything that you are writing. How much money must she still save? How much money must she still save? Hurry up my child. You started very well. You started so well.

Of interest is that while the teacher weakens control by allowing all the learners to finish the task, she also makes regular interjections for the learners to work quickly or 'hurry up'. One could argue that control over pacing is stronger in that the teacher is urging for the task to be completed, however control remains weakened in that the teacher ensures all learners have had the opportunity to complete their working or at least attempt the problem.

Returning attention to the group as a whole, Teacher A affirms that she has 'seen some good plans' and proceeds to allow the selected learners to share their strategies. The teacher ensures that all the learners have stopped working, are attentive and ready to listen to one another's strategies. She assists the learners in their communication to the group by drawing or writing up on the white board what the learner has done in their mat book. Drawing attention to the range of strategies selected, the teacher indicates which strategies are suited to the particular problem and which are most efficient.

Teacher:	I'm going to tell you… I've seen some good plans. I like these two plans. Amila and… Amila what have you used?
Learner:	A number line.
Teacher:	A number line! And what have you used? Alright. Let me quickly show you what Amila did. Amila said… you said I have… ?
Learner:	R45
Teacher:	I have R45! And how much money do you need for the t-shirt?
Learner:	100
Teacher:	100! Amila wants to be here… and she's here *(number line)*. Now what must Amila do to get from here to here? What did you add? Plus?
Learner:	5
Teacher:	Well done. Listen to this. She added 5. 45 plus 5 will give you?
Learner:	50
Teacher:	Well done. And, what must I add to 50 to get to 100?
Learner:	50
Teacher:	So how much money must Amila still save to get to her R100? What have I got? How much must she still save? She must still save this money and this money. How much money is that?
Learner:	55
Teacher:	55. So R45 plus R55 is equal to R100. So what is the answer? My question was …How much must she still save? And your answer is?
Learner:	R55

Control over pacing is further weakened in that Teacher A takes the time to give a number of learners the opportunity to share their solutions, including those she has not selected. She draws out thorough explanations from the learners, questions their understanding of the solution while also reinforcing terminology. Justification for and explication of strategies allows learners to internalise their thinking and make sense of the mathematics.

Learner:	Teacher I did do this.
Teacher:	How did you get it? *(Teacher looks at solution)* Ok so what is 40 plus 5?
Learner:	45
Teacher:	45 plus 10?
Learner:	55
Teacher:	It's also 55. But why did they add 5 first? Why did you add 5 first? She wanted to get to a… ?
All:	A friendly number
Teacher:	And what is a friendly number?
All:	It's a number that has a zero.
Teacher:	And we also call those numbers that end with zero… we have a name for them… we call them… multiples of?

All: 10
Teacher: So you first want to get to a multiple of 10. See, same answer but you just added your amounts… you started with 40 then 5 then 10. She started with 5 and then added 50. Which one do you think was easier? Which one do you think was quicker? Don't you think this was quicker?
All: Yes teacher.
Teacher: But you still have the right answer. Alright.

Framing over pacing in problem-solving tasks is therefore measured F^- in that Teacher A gives all learners the opportunity to attempt the problem and an opportunity to share their strategies with the group. She does, however, maintain a certain degree of control over the rate of transmission. Teacher C displays a similar pedagogy in problem-solving tasks.

Teacher B demonstrates mixed control over pacing in problem-solving tasks ($F^{+/-}$) but tends towards a stronger degree of control overall. The following example highlights Teacher B's strengthening of framing over pacing in problem-solving tasks.

Extract 5C:

Teacher: Work out this answer for me. Um… a farmer plants 12 rows… 12 rows of potatoes… 12 rows of potatoes. In each row there are 5 potato plants. How many potatoes will I get when that is finished… when it's all ready to be eaten? When it's ready to be put in the pot. So how many rows do I have?
Learner: 12 rows
Teacher: How many potato plants in each row?
Learner: 5
Teacher: 5. Right. So what do I have? How many potatoes will I have?

Control over pacing is strengthened in that she does not wait for all of the learners to complete their solutions. While they have their mat books to complete the task, there is little or no time for learners to show their working.

Learner: 60 potatoes
Teacher: How do you know?
Because 12 times 5 is equal to 60.
Teacher: Ok what did you say?
Learner: I said 10 times 5 is equal to 50 and then I said 2 times 5 is equal to 10 and 50 plus 10 is equal to 60.
Teacher: Good right. Very good. She did another method.
Class now open your books. Page 37… is that right?

The first learner shouts out his answer almost immediately, without allowing other learners a chance to attempt the task. While allowing two of the learners to share their strategy, Teacher

B does not elaborate or draw on their strategies used. The strong framing over pacing in this example could also be due to the fact that the task was too easy for the group and did not require working out. Perhaps with a more challenging problem, learners would have the opportunity to spend more time working on their solutions.

In dealing with mixed ability and differential learner pace, Teacher A allocates the same amount of work for all her groups – a page in their NumberSense workbooks and an attribute block activity. Teacher B requires that all her groups complete the allocated pages in their DBE books and their NumberSense workbooks, with the exception of the low-performing group that is required to complete only their DBE pages. Teacher C constructs additional activities so that when learners have completed one activity, they have another with which to engage. An additional shape activity is also allocated to the top-performing group as an extension to keep them occupied while the teacher is on the mat.

In summary, all three of the teachers display stronger control over pacing in counting and manipulating number tasks (F^{+}) while weaker framing in problem-solving tasks (F^{-}). Analysis reveals however, that Teacher B exercises stronger framing over pace in problem-solving tasks than the other two teachers. Teacher B also exercises the least differentiation in the allocation of work to learners.

5.3.3 Evaluative criteria

The following four indicators were used to measure framing over *evaluative criteria*:

In the introduction/exposition to a topic/task

Indicator 1 looks at the extent to which the teacher is explicit or implicit in making the evaluative criteria available to learners at the beginning of the task. It concerns whether the teacher defines or explains concepts, content knowledge or operations required in order to conduct the given task. Where the teacher is most explicit, a score of F^{++} is obtained. A score of F^{--} indicates that little or no attempt is made in terms of the requirements for a successful production of the task.

In selecting learners to show their solutions to a task or activity

Indicator 2 measures a high or low level of selection when asking the learners to share/show their solutions to the task. Where framing is strong (F^{++}), the teacher selects a range of successful strategies from the learners in the group. Where framing is at its weakest (F^{--}), the teacher selects any or no strategies from the learners.

In the kinds of verbal answers required of learners

The third indicator measures the extent to which the teacher requires reasoning or explanation for strategies or solutions to the task. Where framing is strong, the teacher requires substantial reasoning and possible drawing out of principles in supporting their answers. The teacher also elaborates on or corrects solutions provided. Where framing is weak, the teacher looks only for answers without elaboration.

In concluding the task/activity

The fourth indicator refers to the teacher's consolidation or reference to an appropriate production. Where specific comments are made as to what constitutes an appropriate response as well as drawing on examples of success or failure, framing is strong (F^{++}). Where the teacher makes no indication as to what constitutes an appropriate or correct production, framing is considered its weakest (F^{--}).

The following table illustrates the coding of the data in relation to the four indicators for framing over the evaluative criteria. The coding was again measured according to the type of task (counting, mental math or problem solving) carried out by each of the teachers (A, B or C). The teachers' practices are coded according to the indicators, labeled 1- 4, in terms of the strength over framing (F^{--} to F^{++}) in each of the tasks. An aggregate has been found as a means of analysing each teacher's pedagogy as well as any overall patterns that emerge.

Table 5.14: Evaluative criteria in the pedagogic structure

		Counting				Manipulating number				Problem solving			
	Indicator	**1**	**2**	**3**	**4**	**1**	**2**	**3**	**4**	**1**	**2**	**3**	**4**
Teacher A	Group 1	-				F^{+}	F^{-}	F^{--}	F^{+}	F^{--}	F^{++}	F^{++}	F^{+}
										F^{-}	F^{++}	F^{++}	F^{+}
	Group 2	F^{+}	F^{--}	F^{+}	F^{+}	F^{+}	F^{-}	F^{+}	F^{+}	F^{-}	F^{+}	F^{++}	F^{+}
	Group 3	F^{+}	F^{--}	F^{+}	F^{+}	F^{-}	F^{-}	F^{+}	F^{-}	F--	F^{++}	F^{++}	F^{++}
	Trend	F^{+}	F^{--}	F^{+}	F^{+}	F^{+}	F^{-}	F^{+}	F^{+}	$F^{-/--}$	F^{++}	F^{++}	F^{+}
Teacher B	Group 1	-				F^{+}	F^{-}	F--	F^{+}	F^{--}	F^{+}	F^{-}	F^{+}
										F^{--}	F-	F^{-}	F^{-}
	Group 2	-				F^{-}	F^{+}	F^{-}	F^{+}	-			
						F^{-}	F^{-}	F^{+}	F^{+}				
						F^{+}	F^{-}	F^{+}	F^{+}				
	Trend	-				$F^{+/-}$	F^{-}	F^{+}	F^{+}	F^{--}	$F^{+/-}$	F^{-}	$F^{+/-}$
Teacher C	Group 1	F^{+}	F^{--}	F^{+}	F^{+}	F^{+}	F-	F^{+}	F^{+}	F^{--}	F^{++}	F^{++}	F^{++}
						F^{+}	F^{++}	F^{+}	F^{+}	F^{-}	F^{+}	F^{++}	F^{+}
	Group 2	F^{+}	F^{--}	F^{-}	F^{+}	F^{-}	F^{+}	F^{+}	F^{-}	F^{--}	F^{++}	F^{++}	F^{+}
	Group 3	F^{+}	F^{--}	F^{-}	F^{+}	F^{-}	F^{+}	F^{+}	F^{-}	-			
						F^{+}	F^{+}	F^{-}	F^{+}				
	Trend	F^{+}	F^{--}	F^{-}	F^{+}	F^{+}	F^{+}	F^{+}	F^{+}	F^{--}	F^{++}	F^{++}	F^{+}
Overall trend		F^{+}	F^{--}	$F^{+/-}$	F^{+}	F^{+}	F^{-}	F^{+}	F^{+}	F^{--}	F^{++}	F^{++}	F^{+}

In counting tasks, Teacher A and Teacher C display a similar pedagogy in framing over the evaluative criteria. For Teacher A and Teacher C, the requirements for the successful completion of a task are clear and the dominant code is F^{+} for Indicator 1. Framing is then weakened in that neither of the teachers select strategies from the learners, marking Indicator 2 as F^{--}. Framing differs slightly where Teacher A requires reasoning for answers or productions, while Teacher C requires only answers without elaboration. At the end of the task, evaluative criteria are again made explicit by both teachers in consolidating what constitutes an appropriate production, marking Indicator 4 as F^{+}. The example below aims to illustrate this pattern of framing over evaluative criteria in counting tasks for Teacher A and Teacher C.

Teacher A presents the following counting task to her first group. She has been explicit in asking the learners to make five groups with five beans in each group (F^{+}).

Extract 5D:

Teacher:	If you are sitting on the mat waiting there for me, can you please make 5 groups of beans in front of you. Take beans… make 5 groups. Each group must have 5 beans in it.

Where slightly less explicit, while the aim of the task is to count in 25s, the task is set up in such a way that the learners are left to decipher this for themselves.

Teacher:	So now that you have your groups, how many beans do you have? How many beans do you have altogether?
Learner:	75
Teacher:	Count again. How many beans altogether?
Learner:	25
Teacher:	Well done.

The teacher requires only an answer from each of the learners. The task does not allow for a range of strategies as there is no variation, only a correct or incorrect solution (F^{--}).

Teacher:	Well done. [Vusi] is the first one and he says 25 beans. Now, how many beans do you have?
Learner:	25
Teacher:	Alright. Now count on from 25 for me. 25…
Learner:	30; 35; 40; 45; 50.
Teacher:	Alright. So… how many beans have I added to his beans?
Learner:	25
Teacher:	Well done. Well done. Third person. Now, do you need to count or will you be able to tell me. How many beans do you have?
Learner:	25
Teacher:	What must I write here? Count on quickly.
Learner:	75

In terms of verbal responses, the teacher often requires learners to give reasons for their answers and they are also expected to modify their answers until correct (F^{+}).

Teacher:	Ok… what is here? What do you notice? What do you notice here? Do you notice anything?
Learner:	Yes teacher. It's like counting in 25s.
Teacher:	Ja, we are counting in 25s. We are adding 25.
Learner:	We are counting in 5s.
Teacher:	No sweetie pie, we are counting in 25s. 25; 50; 75; 100. So if I add your beans, what will it be?
Learner:	125
Teacher:	Well done. Nice. And if I add your beans, what will this be?
Learner:	150
Teacher:	Well done. How did you get that?
Learner:	125 plus 25 is 150.
Teacher:	125 plus 25 is 150. Ok and if I add your beans... what will this be?

Learner: *(Voice unclear)*
Teacher: No no! It can't be. Count. If you are not sure… 150…
Learner: *(Voice unclear)*
Teacher: No no sweetie pie. Come let's count with Tamia children. Let's help Tamia. She stopped at 150. Count count.
All: 155, 160, 165, 170, 175.

Concluding the task, Teacher A comments on what constitutes a successful production, directed at the group as a whole. Success or failure is indicated before moving on to the next task (F^{+}).

Table 5.15: Framing of counting tasks in evaluative criteria

		Counting			
	Indicator	**1**	**2**	**3**	**4**
Teacher A	Group 1	-			
	Group 2	F^{+}	F^{--}	F^{+}	F^{+}
	Group 3	F^{+}	F^{--}	F^{+}	F^{+}
	Trend	$\mathbf{F^{+}}$	$\mathbf{F^{--}}$	$\mathbf{F^{+}}$	$\mathbf{F^{+}}$
Teacher B	Group 1	-			
	Group 2	-			
	Trend	**-**			
Teacher C	Group 1	F^{+}	F^{--}	F^{+}	F^{+}
	Group 2	F^{+}	F^{--}	F^{-}	F^{+}
	Group 3	F^{+}	F^{--}	F^{-}	F^{+}
	Trend	$\mathbf{F^{+}}$	$\mathbf{F^{--}}$	$\mathbf{F^{-}}$	$\mathbf{F^{+}}$
Overall trend		$\mathbf{F^{+}}$	$\mathbf{F^{--}}$	$\mathbf{F^{+/-}}$	$\mathbf{F^{+}}$

Teacher C varies only slightly in some counting tasks in that learners are rarely required to give reasons for their answers. Framing is therefore marked weaker (F^{-}) in requiring justification for responses. This variation between Teacher A and Teacher C could also stem from the varied complexity of the tasks. Teacher C, for example, presents a task of counting beans in groups of ten. At a Grade 3 level, this is fairly straight forward and perhaps requires less discussion than counting in 25s. Teacher B is not coded in this instance as she practices counting as a whole class activity.

In manipulating number tasks, the three teachers display a similar pedagogy. In most instances they are explicit as to how the task should be completed (Indicator 1: F^{+}), with the exception of a few tasks where Teacher B is less clear. Similar to counting tasks, selection based on strategies holds weak framing for all teachers in that while most learners are given the opportunity to answer questions, a single response rather than a range is sought (Indicator 2: F^-). Where Teacher C varies slightly is that the tasks she selects lend themselves to a greater range of strategies, potentially strengthening the framing of evaluative criteria (F^+). For all teachers, learners are to a certain extent asked to verbalise and justify their thinking and the teacher elaborates on a correct or incorrect response (Indicator 3: F^+). At the end of the task, each teacher clearly elaborates on what constitutes an appropriate response (Indicator 4: F^+).

The following extract serves as an example:
Having completed a task of counting in 11s, Teacher C continues with a manipulating number task around multiples of 11. The learners are clear as to the criteria of the task (Indicator 1: F^+).

Table 5.16: Framing of manipulating number tasks in the evaluative criteria

Extract 5E:

Teacher:	What is 12 times 11?
Learner:	121
Learners:	Nooooooo!
Teacher:	Ok, just relax. What is 12 times 11? What is 10 times 11? Just give him a chance. Will you sit down? What is 10 times… um … 10 times 11?
Learner:	110
Teacher:	Right. So what is 11 times 11?
Learner:	121
Teacher:	So what is 12 times 11?
Learner:	132
Teacher:	Very good. Can you explain your answer? How did you get your answer so quick?
Learner:	I said teacher, 110 +11 is equal to 121. Then I said 10 + 121 is equal to 131 and then there is 1 left… is 132
Teacher:	Is there an easier way? What's the easiest way?
Learner:	11 times 11 is equal to 121, plus 11 is equal to 132.
Teacher:	Very good. Thank you.

		Manipulating number			
	Indicator	**1**	**2**	**3**	**4**
Teacher A	Group 1	F^+	F^-	F^{--}	F^+
	Group 2	F^+	F^-	F^+	F^+
	Group 3	F^-	F^-	F^+	F^-
	Trend	$\mathbf{F^+}$	$\mathbf{F^-}$	$\mathbf{F^+}$	$\mathbf{F^+}$
Teacher B	Group 1	F^+	F^-	F--	F^+
	Group 2	F^-	F^+	F^-	F^+
		F^-	F^-	F^+	F^+
		F^+	F^-	F^+	F^+
	Trend	$\mathbf{F^{+/-}}$	$\mathbf{F^-}$	$\mathbf{F^+}$	$\mathbf{F^+}$
Teacher C	Group 1	F^+	F-	F^+	F^+
		F^+	F^{++}	F^+	F^+
	Group 2	F^-	F^+	F^+	F^-
	Group 3	F^-	F^+	F^+	F^-
		F^+	F^+	F^-	F^+
	Trend	$\mathbf{F^+}$	$\mathbf{F^+}$	$\mathbf{F^+}$	$\mathbf{F^+}$

The teacher moves swiftly through the task as it does not lend itself to a range of strategies (Indicator 2: F^-). Teacher C does, however, draw reasoning from the learners, asking how their solutions were acquired (Indicator 3: F^{+}). The teacher also corrects learners and elaborates on what constitutes an appropriate response (Indicator 4: F^{+}).

Manipulating number tasks tend to be an entrenched practice where overall framing of evaluative criteria is strong (F^{+}). Instances of weakened framing tend to occur only when the task does not lend itself to a range of strategies to be selected.

In problem-solving tasks, Teacher A and Teacher C display similar pedagogies in framing over evaluative criteria. The coding of the data across the four indicators shows the general pattern: $F^{-/--}$; F^{++} ; F^{++} ; F^{+}. This suggests a different trajectory in the pedagogy when it comes to problem solving, where framing shifts from weak; to very strong control; to strong control as these teachers work through the task with the learners. The example below aims to illustrate this.

Teacher A begins by presenting the learners with a problem. She provides no indication as to how the problem should be solved in terms of a method or calculation strategy, but simply states that they should 'make a plan'. Framing over the exposition to the task is therefore classified as weak (Indicator 1: F^{--}).

Extract 5F:

Teacher: You know Mr Henchel gives us a box of books every term, but at the end of the term I must give him all the books back. Ok… so… Mr Henchel gave us a box with 127 books. Do you know how many books? Mr Henchel gave us a box and in the box there were 127 books. Now I must return the books to him and I've counted the books and I don't have 127. I have counted only 95 books. Can you tell me how many books I must still find? Some of you must still return the books.
How many children must still return the books so that I can give all the books back to Mr Henchel. Show me your plan.

While working out the problem in their mat books, Teacher A walks around the group, deliberately looking for different strategies or responses which have been successful. Having selected two strategies from the group, the teacher then asks the learners to put down their pencils and listen to the different responses. Framing is strong (Indicator 2: F^{++}) in that the teacher is deliberate in selecting a range of successful strategies from the group.

Teacher A begins with the first learner's approach, facilitating discussion by asking questions and making use of the white board to prompt thinking.

Teacher: [Mary] says… listen to this… she says I must give Mr Henchel 127 books. Open let me see. *(Points to mat book)* And then she said I already have 95 books so she takes away, she subtracts the books that I have, so that she can calculate how many books I must get. This is what she did. I want to show you what she did.
She said 12 minus 9 is…. And what is your answer? *(Teacher draws on white board)*

$$127 - 95 = 3$$

Learner: 3
Teacher: She said 12 minus 9 is 3. And then she said 7 minus 5 is?

$$127 - 95 = 32$$

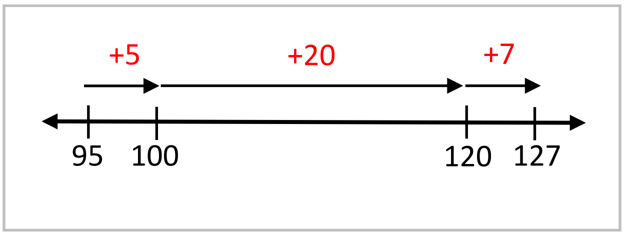
Learner: 2
Teacher: That is how she got an answer of 32.

A second child interjects, eager to share his idea and while providing a method and solution that is correct, Teacher A does not elaborate with the rest of the group. Reasons for this could be that the method would cause confusion for the other learners.

Learner: I said 100 minus 90 is 10 and 20 minus 5 is 15…that is 25. And then I added the 7 to get 32.
Teacher: Did you also get 32?
Learner: *(Learner nods)*
Teacher: Alright.

The 'selected' second child then proceeds to share her strategy of using a number line. The teacher also assists the learner by illustrating what has been done on the whiteboard.

Teacher: What have you done?
Learner: I used a number line.
Teacher: Number line! Aha…right! Come show me your number line. See what happened on your number line. I like this number line very much. Well done. *(The teacher draws a representation of what the learner did in her book).*



Teacher:	[Thandi] said… we have 95 books… then [Thandi] added 5. That is what she did. You see. And what will this be?
Learner:	100
Teacher:	100. And then [Thandi] said… plus 20.
Learner:	Is 120
Teacher:	This is 120. And…. Where do I want to be… 127… so what must I still add?
Learner:	Plus 7
Teacher:	Plus 7. Ok let's do this. 5 plus 20 is?
Learner:	25
Teacher:	And 25 plus 7?
Learner:	32
Teacher:	Same answer hey.

Framing is therefore strong (Indicator 3: F^{++}) in the kinds of verbal responses the teacher requires from the learners. She deliberately draws upon the successful approaches used and encourages learners to verbalise their thinking. This intends not only to assist the learner selected but also equips other learners for future productions. She also does not elaborate on potentially distracting or confusing responses.

The teacher then offers a third solution created by a learner 'in another class'. While this is fictional, rather than providing a fixed method of her own the teacher is deliberate in attempting to guide the group towards a faster and more efficient strategy.

Teacher:	Can I show you something that I've seen someone do. Can I show you? Can you put your pencil down. Somebody who's done the same calculation said… I just want to see if you think this person used a good strategy. It's all you have to tell me. This person said … I think it was pretty clever. You can see if you agree with me. He said… 127 minus 100 …what is that?
Learner:	27
Teacher:	Right. Can you tell me… this and this… what is that person doing?
Learner:	Teacher I know. She did plus 5.
Teacher:	And if I say 27 plus 5… what is that? Did we all get to the same answer? I'm not going to tell you but it was someone that was in Grade 3 last year. Do you think it's a clever idea? Would it work for you? Ok. See if you can do this in your book for me.

By providing an alternative 'fictional' strategy, which is of course her own, the teacher deliberately masks her role as teacher and holder of appropriate criteria. This is taken up in the framing of hierarchical rules which follows. Framing is therefore slightly weakened (Indicator

4: F^{+}) in concluding the task. Teacher A builds on this strategy by providing an additional problem as consolidation. This follows the same context, only that the number range has now been increased. The teacher encourages learners to consider the strategies used in the previous problem. While the teacher does not obtain the approach suggested, she also does not insist on it and allows learners again to share their ideas with the group.

Teacher:	In the Grade 5 class they get many more books hey? They get a big box. They get 241 books. 241 books. And… she only collected 198 books. And she must give Mr Henchel 241 books. And he only collected 198 books. How many books must she still collect? Now look at all your strategies.
Learner:	Teacher must we use this one?
Teacher:	No no. You can use any method. I just gave you some more options to work with. Do you have an answer for me? Make a plan. What is your answer? I can see your answer. Mr Fontein had 241 books in her box. She only collected 198. There are still books outstanding. How many learners must still return the books? I like your thinking… but… just check again. Remember we had a few plans here.

Teacher C displays a similar approach in problem-solving tasks and begins by presenting her learners with the following problem:

Extract 5G:

Teacher:	Now remember next week we are going to the school of …
Learners:	Magic
Teacher:	Magic. Ok. Alright. We're going to the school of magic… and how many children are we in the class?
Learner:	27
Teacher:	27. And the school of magic charges …
Learner:	R5
Learner:	R10
Teacher:	No, no! Excuse me! The school of magic charges R9 …no no no. The school of magic charges R6 per child. Ok. How much will we pay altogether? You need to remember... that is the problem you are going to do in your book. The school of magic charges R6, alright? And how many children in our class?
Learners:	27

Teacher C leaves the learners to solve the problem without elaborating on an appropriate response or approach (Indicator 1: F^{--}). As the learners complete their solutions, they hand their mat books to the teacher to show what they have done. The teacher takes the time to select a range of successful strategies to share with the group. Framing is therefore strong (Indicator 2: F^{++}) in that she is deliberate in her selection. She also assists those learners who have made errors, asking them to reconsider their working out.

Teacher C then asks the selected learners to share their strategies on the board and facilitates the discussion around what they have done. Learners are asked to elaborate on their thought processes and others are encouraged to engage and consider which of the strategies is most efficient. Framing over the responses required is then also strong (Indicator 3: F^{++}) in that Teacher C draws this out in great detail.

Teacher:	Can you see what he did? What he did was he took the 6 and he times'd it by 27 but he times'd it by 5 and by 5and by 5 and by 5 and 5 and 2.
Learner:	First I wrote 6 times 27 is equal to…. Then I did write 6 times 20 is equal to 120. 6 times 7 is equal to 42. 120 plus 42 is equal to 162. So 6 times 27 is equal to 162.
Teacher:	Which way do you think is the quickest way to work it out? Which way?
Learner:	[Iviwe's]
Teacher:	Right. [Iviwe's] way is the quickest way to work it out right.

Finally, Teacher C leads the learners towards working with the most efficient strategy and proceeds to offer a similar problem for them to solve. Framing is strong (Indicator 4: F^{++}) in the selection of strategies and reasoning behind approaches.

As a whole, Teacher A and Teacher C display very weak framing (F^{--}) in the introduction to problem-solving tasks. This is then contrasted with very strong framing (F^{++}) when both teachers select specific strategies as well as elicit and elaborate on verbal responses (F^{++}) from the learners. Framing remains strong ($F^{+/++}$) in conclusion to the task in that both teachers acknowledge successes and failures and make comments and suggestions as to which strategies are most efficient. Teacher A displays slight weakening in framing as she does not insist on a particular approach.

Teacher B demonstrates a slightly different approach in that framing over the evaluative criteria remains for the most part weak, shifting only when concluding the task. This is indicated in the example below.

Extract 5H:

Teacher:	Work out this one for me quickly. Um… a farmer plants …12 rows… 12 rows …of potatoes… 12 rows of potatoes. In each row there are 5 potato plants. How many potatoes will I get when that is finished… when it's all ready to be eaten? Ready to be put in the pot. So how many rows do I have?
Learners:	12 rows
Teacher:	How many potato plants in each row?
Learners:	5
Teacher:	5. Right, so what do I have?

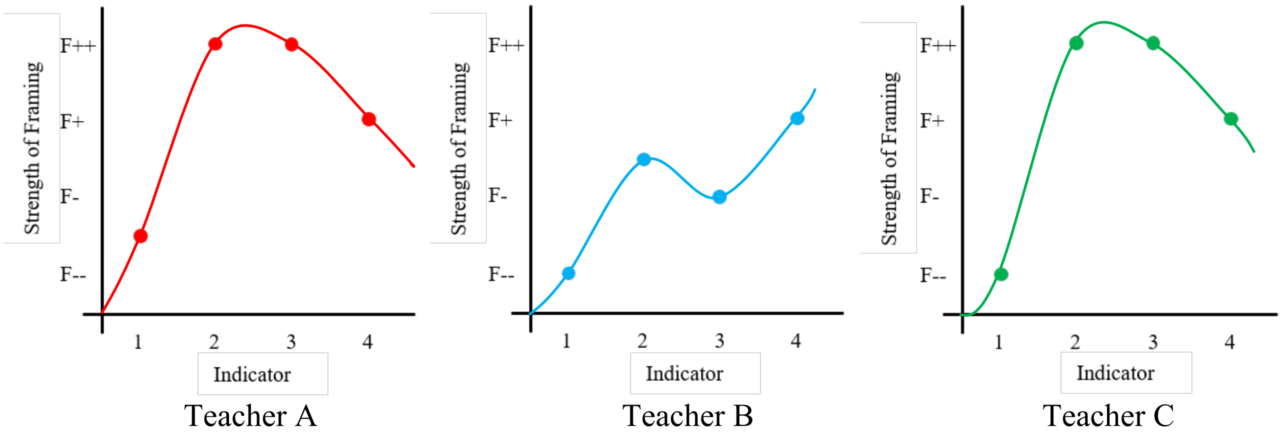
The teacher begins by introducing the problem to the learners without drawing out knowledge principles or any indication as to how the task should be completed. The teacher is implicit in the requirements for an appropriate production (Indicator 1: F^{--}). When learners raise their hands almost immediately, the teacher selects any of the learners to share their strategy/answer with the group.

Teacher:	How many potatoes will I have?
Learner:	60 potatoes
Teacher:	How do you know?
Learner:	Because 12 times 5 is equal to 60.
Teacher:	Ok. What did you say? *(Directed at a different learner)*
Learner:	I said 10 times 5 is equal to 50 and then I said 2 times 5 is equal to 10 … and 50 plus 10 is equal to 60.
Teacher:	Good! Right… very good! She did another method. Class now open your books.

By randomly selecting two of the children to share their strategies, Teacher B demonstrates a reasonably low level of selection (Indicator 2: F^{-}). When sharing their strategies, these are accepted and to some extent praised but not elaborated on… 'Right…very good! She did another method'. Framing is then relatively weak in terms of the kinds of verbal answers required of the learners (Indicator 3: F^{-}) and only slightly strengthened in the validation that the answers are correct (Indicator 4: F^{+}). The other learners are also not further guided as to which of the strategies are favoured or more efficient. The solutions from the rest of the group are not monitored or commented on.

Through analysis of the three teachers in terms of problem-solving tasks, while the data sample is small, one begins to notice clear differences in the pedagogy of Teacher A and Teacher C in comparison to that of Teacher B.

Figure 5.2: Patterns revealed in evaluative criteria



In the introduction or explanation to a task, all three of the teachers display weak framing in seldom indicating how the task should be completed, and not revealing the mathematical knowledge required to complete the task. Initiation of the task and the strategies used are left open for the learners to interpret and administer in their own way.

Having completed the task however, Teachers A and C begin to take a different path to that of Teacher B. Teacher A and Teacher C take a greater lead in selecting strategies used by the learners and which should be presented to the group. These teachers make their selection based on successful strategies as well as looking for a range of different approaches. While Teacher B selects learners to share their solutions, she does not make her selection based on learners' productions. Teacher A and C then display a higher level of control over framing than that of Teacher B.

In the verbal responses provided by the learners, Teacher A and C require a strong degree of reasoning behind strategies used and discussion around how they came about their strategy. Both teachers prompt the learners and ask questions to facilitate the discussion, enabling the learners to explain their thought processes. For Teacher B however, while the children do explain their strategies, she does not elaborate on their production or draw out specific knowledge principles.

Finally, all teachers are fairly explicit in sharing the successes in the group, displaying what constitutes an appropriate production as well clarifying where learners may have gone wrong. None of the teachers, however, insist on a particular strategy or method to be used.

Taking into account the types of tasks played out in each of the groups –counting, manipulating number and problem-solving tasks – one begins to notice certain patterns in terms of the evaluative criteria. In counting and manipulating number tasks, control over the evaluative criteria remains for the most part strong, weakening only in terms of the strategies used. In contrast, for Teacher A and Teacher C, framing over the evaluative criteria starts off weak in problem-solving tasks and displays gradual tightening or strengthening as the task is played out, suggesting a deliberate trajectory in evaluative criteria.

Teacher B is reluctant to weaken framing, displaying a preference to whole class teaching over differentiated teaching and does not easily lean towards individualising learners during tasks. This was confirmed in an interview with Teacher B.

5.3.4 Hierarchical rules

The following four indicators were used to measure framing over *hierarchical rules*:

In facilitating discussion in the group

The first indicator considers the extent to which the teacher directs the discussion in the group and her attempts in understanding learners' thought processes. Strong framing indicates that the teacher provides little or no opportunity for learners to share their thought processes. Weak framing indicates that the teacher allows the learners to speak freely and makes a good attempt to understand their thinking.

In presenting solutions to the group

The second indicator relates to the way in which the teacher allows learners to demonstrate their solutions to the group. Framing is considered strong when learners demonstrate their solutions only to the teacher and the other learners do not participate or engage in the presentation. Weak framing indicates that learners demonstrate and share their thinking with the whole group and the other learners are actively engaged in the presentation.

In communication relations between the teacher and the learners in the group

The third indicator refers to the extent to which the teacher limits or allows communication within the group. Where framing is strong, the teacher closes teacher-learner communication and learners participate only when invited to by the teacher. Where framing is weak, the teacher allows and encourages open communication and learners are free to initiate or respond to interactions in the group.

In the rapport between the teacher and learners in the group

The final indicator considers the extent to which the teacher displays friendliness or openness to the learners in the group. Where framing is strong, the teacher displays no friendliness or offers no words of praise. Where framing is weak, the teacher is informal and open with the group, promoting discussion between learners in the group. The table below illustrates coding of framing over hierarchical rules for each of the teachers.

Table 5.17: Hierarchical rules in the pedagogic structure

		Counting				**Manipulating number**				**Problem solving**			
	Indicator	**1**	**2**	**3**	**4**	**1**	**2**	**3**	**4**	**1**	**2**	**3**	**4**
Teacher A	Group 1	-				F^{+}	F^{+}	F^{+}	F^{-}	F^{--}	F^{--}	F^{--}	F^{--}
										F^{--}	F^{--}	F^{--}	F^{--}
	Group 2	F^{--}	F^{-}	F^{--}	F^{-}	F^{-}	F^{-}	F^{-}	F^{-}	F^{--}	F^{-}	F^{--}	F^{--}
	Group 3	F^{--}	F^{-}	F^{-}	F^{-}	F^{-}	F^{-}	F^{-}	F^{-}	F^{--}	F^{--}	F^{--}	F^{--}
	Trend	$\mathbf{F^{--}}$	$\mathbf{F^{-}}$	$\mathbf{F^{-/--}}$	$\mathbf{F^{-}}$	$\mathbf{F^{-}}$	$\mathbf{F^{-}}$	$\mathbf{F^{-}}$	$\mathbf{F^{-}}$	$\mathbf{F^{-}}$	$\mathbf{F^{-}}$	$\mathbf{F^{-}}$	$\mathbf{F^{-}}$
Teacher B	Group 1	-				F^{+}	F^{+}	F^{+}	F^{-}	F^{-}	F^{+}	F^{+}	F^{-}
										F^{+}	F^{-}	F^{-}	F^{-}
	Group 2	-				F^{-}	F^{-}	F^{-}	F^{--}	-			
						F^{-}	F^{-}	F^{-}	F^{-}				
						F^{-}	F^{-}	F^{--}	F^{-}				
	Trend	**-**				$\mathbf{F^{-}}$	$\mathbf{F^{-}}$	$\mathbf{F^{-}}$	$\mathbf{F^{-}}$	$\mathbf{F^{-}}$	$\mathbf{F^{+/-}}$	$\mathbf{F^{+/-}}$	$\mathbf{F^{-}}$
Teacher C	Group 1	F^{-}	F^{-}	F^{-}	F^{-}	F^{-}	F^{-}	F^{-}	F^{--}	F^{--}	F^{-}	F^{--}	F^{--}
						F^{--}	F^{--}	F^{-}	F^{--}	F^{--}	F^{-}	F^{-}	F^{--}
	Group 2	F^{-}	F^{-}	F^{-}	F^{--}	F^{--}	F^{-}	F^{-}	F^{--}	F^{--}	F^{--}	F^{-}	F^{--}
	Group 3	F^{--}	F^{-}	F^{-}	F^{-}	F^{-}	F^{--}	F^{-}	F^{-}	-			
						F^{-}	F^{-}	F^{-}	F^{--}				
	Trend	$\mathbf{F^{-}}$	$\mathbf{F^{-}}$	$\mathbf{F^{-}}$	$\mathbf{F^{-}}$	$\mathbf{F^{-}}$	$\mathbf{F^{-}}$	$\mathbf{F^{-}}$	$\mathbf{F^{--}}$	$\mathbf{F^{--}}$	$\mathbf{F^{-}}$	$\mathbf{F^{-}}$	$\mathbf{F^{--}}$
Overall trend		$\mathbf{F^{-}}$	$\mathbf{F^{-}}$	$\mathbf{F^{-}}$	$\mathbf{F^{-}}$	$\mathbf{F^{-}}$	$\mathbf{F^{-}}$	$\mathbf{F^{-}}$	$\mathbf{F^{-}}$	$\mathbf{F^{--}}$	$\mathbf{F^{--}}$	$\mathbf{F^{-}}$	$\mathbf{F^{--}}$

For all three of the teachers, framing over hierarchical rules is defined by more personal control in the teachers' facilitation of discussion as well as the learners' presentation of solutions to the group. The teachers are open and friendly and, in most instances, encourage the learners to speak freely in the attempts to better understand thought processes. Overall, framing of the hierarchical rules is therefore coded as weak (F^-). The following analysis aims to provide examples of this as well as instances where coding differs among teachers or certain tasks.

In counting tasks, Teacher A and Teacher C display weak framing overall (F^-). Both teachers ensure learners' understanding of the task by facilitating discussion and allowing learners to share their thinking. This is at times limited due to the nature of the task and the intention to promote fluency through pace rather than detailed discussion. The teachers create an environment where learners are free to interact with one another around the topic of mathematics. Again, Teacher B is not measured in terms of counting tasks as this was completed as a whole class activity.

In manipulating number tasks, framing holds weak (F^-) for all three of the teachers with the exception of a few instances where, again, the task does not lend itself to discussion

Table 5.18: Framing of manipulating number tasks in hierarchical rules

		Manipulating number			
	Indicator	**1**	**2**	**3**	**4**
Teacher A	Group 1	F^+	F^+	F^+	F^-
	Group 2	F^-	F^-	F^-	F^-
	Group 3	F^-	F^-	F^-	F^-
	Trend	$\mathbf{F^-}$	$\mathbf{F^-}$	$\mathbf{F^-}$	$\mathbf{F^-}$
Teacher B	Group 1	F^+	F^+	F^+	F^-
	Group 2	F^-	F^-	F^-	F^{--}
		F^-	F^-	F^-	F^-
		F^-	F^-	F^{--}	F^-
	Trend	$\mathbf{F^-}$	$\mathbf{F^-}$	$\mathbf{F^-}$	$\mathbf{F^-}$
Teacher C	Group 1	F^-	F^-	F^-	F^{--}
		F^{--}	F^{--}	F^-	F^{--}
	Group 2	F^{--}	F^-	F^-	F^{--}
	Group 3	F^-	F^{--}	F^-	F^-
		F^-	F^-	F^-	F^{--}
	Trend	$\mathbf{F^-}$	$\mathbf{F^-}$	$\mathbf{F^-}$	$\mathbf{F^{--}}$

This is evident with Teacher A and Teacher B where the pace and the procedural nature of the task requires only solutions. The examples highlighted consisted of a 'bonds of 20' and a 'doubling and halving' activity, neither of which required discussion or reasoning. The tasks chosen by Teacher C, however, lend to greater discussion and validation. While open and friendly, the three teachers maintain a more rigid structure in keeping a pace where fluency is emphasised.

Framing shifts again in problem solving where the tasks lend themselves to discussion and reflection. Presented with tasks that allow for a number of different approaches, the potential for interaction is far greater. Again, there is only slight variation between Teacher A and Teacher C who, in most cases, reflect very weak framing (F^{--}). Teacher B maintains a stronger level of control (F^{-}) in communicative interactions within the group. Instances of this are illustrated in the examples below.

Teacher C presents the following problem to the group. She includes the learners in the conversation by allowing interjections and contributions to the context of the problem.

Extract 5I:

Teacher: Now remember next week we are going to the school of …
Learners: Magic
Teacher: Magic. Ok. Alright. We're going to the school of magic… and how many children are we in the class?
Learner: 27

Teacher: 27. And the school of magic charges …
Learner: R5
Learner: R10
Teacher: No, no! Excuse me! The school of magic charges R9 …no no no. The school of magic charges R6 per child. Ok. How much will we pay altogether? You need to remember... that is the problem you are going to do in your book. The school of magic charges R6, alright? And how many children in our class?
Learners: 27

The learners begin working in their mat books and the teacher takes the time to look at each of the learners attempts or strategies before making her selection. When she is sure that everyone has completed the task, the first selected learner is asked to share his strategy by writing on the white board held by the teacher.

6 x 5 = 30
6 x 5 = 30
6 x 5 = 30
6 x 5 = 30
6 x 5 = 30
6 x 2 = 12
= 162

When the learner has written the above, the teacher reinforces what he has done by asking other learners to describe what he did and further, by explaining it back to the group herself.

Teacher: Right. Give him a clap. Right. Thank you! So can you see what he did? Who can explain? Do you know what he did? What did he do? What he did is he took the 6 and he times'd it by 27 but he times'd it by 5 and by 5 and by 5 and by 5 and 5 and 2.

The second child is then invited to share his strategy.

$6 \times 27 = 162$ (filled in at end)
$6 \times 20 = 120$
$6 \times 7 = 42$
$120 + 42 = 162$.
Iviwe

While showing his working on the whiteboard, the learner also verbalises his thinking.

Learner: First I wrote 6 times 27 is equal to… Then I did write 6 times 20 is equal to 120. 6 times 7 is equal to 42. 120 plus 42 is equal to 162. So 6 times 27 is equal to 162.

The teacher then asks the group to decide which of the methods is most efficient.

Teacher: Which way do you think is the quickest way to work it out? Which way?
Learner: [Iviwe's]
Teacher: Right. [Iviwe's] way is the quickest way to work it out right.

Overall, Teacher C allows learners to speak freely as a means of understanding their thought processes (F^{--}). Learners present and explain their solutions to the whole group, however there are instances where not all of the learners are engaged (F^{-}). Open teacher-learner communication is encouraged (F^{--}), where the teacher allows an open and friendly environment and motivation is promoted (F^{--}). The same holds true for Teacher A who successfully engages all of the learners in the discussion, marking framing weaker for indicator 2 (F^{--}). As a means of consolidation, Teacher C also provides the group with a second problem, encouraging them to use Iviwe's method.

Teacher: Now I'm just going to give you one last sum and you are all going to use Iviwe's way, alright? You're all going to use Iviwe's way. Right. Or you're going to try.
We are having a Heritage Day tomorrow, right? Tomorrow which is Friday. They are going to sell…listen to me… they are going to sell popcorn… are you listening to me? Are listening to me? They are going to sell popcorn at….um… let me see… at R4 a packet. It's a big packet of popcorn. They worked that in the grade 4 class alone, 44 children ordered popcorn. How much money did they pay? While you're busy with that… remember we're going to use Iviwe's method. I'm coming to check up on the rest of you *(learners seated at their desks)*…

Having selected two strategies, the teacher again asks for learners to demonstrate to the group.

The first selected learner writes the following solution on the board and proceeds to explain his thinking.

$40 \times 4 = 160$
$4 \times 4 = 16$
$160 + 16 = 176$

Learner: I did say 40 times 4… I don't know what that is, so I said that 4 times 4 is equal to 16 and I add a 0, then I got 160. Then I got 4 times 4 is equal to 16. Then I said that 160 plus 16 is equal to 176.
Teacher: Is she right?

Learners: Yes teacher.
Teacher: Now do you know what I like? She said she didn't know what 40 times 4 was but she knew what 4 times 4 was… is 16. Ok? So that's how she got 160.
Now open your books…

Teacher C asks for consensus from the other learners in the group as a means of engaging everyone in the presentation. The teacher also highlights and encourages the learner's methodology of adding a zero when working with multiples of 10.
She then proceeds to ask the second learner to present and explain her strategy to the group.

$2 \times 44 = 88$
$2 \times 44 = 88$
$80 + 80 = 160$
$8 + 8 = 16$
$160 + 16 = 176$

Learner: I know that 2 times 44 is equal to 88. Then I said 2 times 44 is equal to 88. Then I said I know 80 plus 80 is equal to 160. 8 plus 8 is equal to 16. Then 160 plus 16 is equal to 176.

By deliberately selecting two successful strategies, the teacher's intention is to provide other learners in the group with tools for future problem solving. While the intention is for the learner to present to the teacher and the whole group, by this stage there are only a few learners who are actively engaged, shifting the indicator from F^{--} to F^{-}. This is perhaps because the group has spent too long on the mat and most learners have lost concentration.

Teacher A displays a similar pedagogy as coding is weak (F^{-}) for hierarchical rules. The teacher does, however, tend to mask her role as teacher, focusing on the learners' productions and ensuring that they are shared amongst the group. Returning to the problem discussed earlier with '*Mr Henchel and his books',* the teacher presents a 'fictional' strategy, saying that it belonged to another learner rather than herself. Teacher A is seen to blur the boundaries between teacher and learner control, perhaps as a means of empowering the learners to 'solve' problems independently. This relates to Vygotsky's notion of invisible mediation where the requirements for completing the task are implicit yet there is an underlying motivation to use a particular strategy. While suggesting an invisible pedagogy, there is a shift to visible at the point of indicating what constitutes as an appropriate response to the task.

Framing is also weakened ($F^{+/-}$) with Teacher B but not to the extent of the other two teachers (F^{--}). Teacher B presents the group with the problem below.

Extract 5J:

Teacher:	There was a tuck shop up during interval… it was Market Day… and these were the prices that were on the wall. A packet of chips was R3. Quickly… A packet of chips …R3. A hotdog was R15 each… a hotdog was R15 each. And I got so thirsty I needed a cooldrink to drink... and so… the cooldrink's were R5 each. I thought it was expensive but it was Market Day and it was fundraising. The cooldrinks were… what did I say?
Learners:	R5
Teacher:	R5 each. Right. Now that was for how many items? How many items have you got on your list?
Learners:	3
Teacher:	3. And was that the price for 5 or was that just for one?
Learner:	One
Teacher:	One. Ok. I bought… this is what I bought at interval…Write down what I bought. I bought 3 packets of chips. I bought … I wasn't gonna eat it all right… I was going to share with some children who didn't have money. Um … I bought 4 hotdogs and I bought 2 cooldrinks. Right. Work out for me quickly what that came to and then I'll give you the next part of the sum. First tell me what that came to. You must able to explain how you got the answer also hey.

Teacher B allows the learners time to complete the problem in their mat books. She observes that some of the learners are finished but does not monitor different strategies used. At this stage she is more interested in a solution rather than a strategy.

Teacher:	I want to see the amount… the total… what it came to. I want to see the total what it came to. Some people have it, some don't. Ok. Right. You done? Hands up. Right, put up your hand… who has the total for me? Some people are still working it out. Ok will Leo and … is still thinking? Ok Leo… tell me… what total do you get for all the lunch things that I bought?
Learner:	79
Teacher:	You have 79. Anybody else get a different number? Did you all get 79?
Learners:	Yes teacher.
Teacher:	Good. Then its right.

While Teacher B asks one of the learners for their solution, she does not elaborate on his strategy but rather gauges with the other learners whether the answer is correct or not. Having ascertained that the answer is in fact correct, she then asks a second learner to explain his strategy.

Teacher:	Let's see if we right. Explain [Alutho]… to the group… how you got your answer.
Learner:	3 times 3 is 9. Then I wrote 15 times 4 is equal to 60. Then I have … 5 times 2 is 10
Teacher:	Add it together and then you got…
Learner:	79

Teacher B demonstrates slight shifting between positional and personal control. While asking the necessary questions to facilitate discussion, she is also quite specific in the kinds of responses she would like. She shows greater comfort in keeping positional boundaries clear. This is evident from the beginning of the lesson when the teacher commences with a whole-class counting activity. Here the teacher conducts from the front of the class, holding a greater level of control over interactions. While the teacher partakes in the group routines, one is given the impression that this is not part of her regular routine.

For Teacher A and Teacher C, discussion and reasoning play a dominant role, especially in problem-solving tasks. Through controlled mediation within the group, the teacher is able to facilitate thought processes and guide discussion around strategies while at the same time developing conceptual understanding.

5.4 Conclusion

In describing the pedagogic structure of the three teachers in the data sample, the intention has been to look at interpretations and instances of differentiated instruction through a number of frameworks for analysis. Through consideration of both the grouping of learners and control relations in the framing of the pedagogic structure, subtle differences and particular aspects of the differentiating pedagogies become apparent.

While all three teachers engage in differentiated teaching, one comes to realise the different forms it takes. Firstly, there is evidence of shifting control relations between the different teachers in the data sample. Secondly, there are shifts in control relations as different tasks are played out. What remains for the most part unchanged however, is variation between the different groups across all three of the teachers. While there is slight variation in content between groups as one might expect, the primary shift in control relations is related to the tasks and this varies between teachers but not between groups.

A central aspect is the discussion and reflection around tasks, especially in terms of problem solving. The teacher-learner relationship is one which is collaborative where, through the guidance of the teacher, learners are mediated towards newly acquired knowledge. While Teacher A and Teacher C display a definite differentiated teaching practice, Teacher B presents a pedagogy which dabbles with differentiation *within* a homogenous teaching practice. Of most significance from the analysis is the operation of control relations around the evaluative criteria and hierarchical rules in the data sample. The evidence of a trajectory suggests a framework through which mediation can be measured and how it functions in the development of mathematical understanding.

Chapter 6: Discussion and conclusion

6.1 Introduction

In this study, I have set out to explore differentiating practices within the context of an early-grade mathematics intervention. By means of a qualitative study, my intention has been to illustrate interpretations of differentiated instruction and in this discussion, I also suggest approaches that potentially have greater impact on learners' mathematical proficiency. Situating mediation as a central feature to differentiated instruction, I have explored the productivity of a Bernsteinian framework through which mediation within differentiation can be measured systematically. Through control relations in the selection, sequence, pace, evaluative criteria and hierarchical rules within the pedagogic structure, I consider the extent to which "framing refers to the interactional aspects of pedagogy" and how these relations influence teaching and learning (Hoadley & Muller, 2010, p. 71). The data sample reveals how mediation 'happens' in different ways and how one can begin to look toward instances or patterns which influence the potential for learning.

6.2 The role of mediation

Behind Vygotsky's (1978) *socio-cultural theory* is the claim that teaching and learning is a socially mediated activity. Central to mediation, is the shared understanding and interaction between the teacher and learners through a language that can later be internalised into higher mental function. For effective mediation to take place, I have taken two key features into consideration. Firstly, mediation requires grouping of learners according to levels of development in mathematics as well as the organisational form within the mathematics lesson. Secondly, mediation calls for cognisance of the nature of control relations in the pedagogic structure. This is of most significance in the framing of evaluative criteria and hierarchical rules and is the point at which mediational patterns begin to emerge. These features are elaborated in greater detail below.

6.2.1 Grouping learners

Analysis of the observation data shows that Teacher A and Teacher C display a stronger sense of routine and structure than Teacher B in terms of a differentiated approach to teaching and learning. This is evident in the comparison of results between the diagnostic assessment and teacher grouping as well as the organisation in terms of the instructional form within each of the classrooms. Teacher A and Teacher C indicate a clear awareness of ability of the learners in each group. These groups show similar levels of development in mathematics and, according to interviews, are adjusted regularly. The learners are familiar with group-work routine and the rotation of the groups is seamless. Due to infrequency on the mat, Teacher B is less 'in tune' with the nature of her groups, the number of learners and the levels of development within each group. The learners are not evenly allocated across the groups and the teacher is unclear as to which learners belong in each group. The differentiating pedagogy presented on the day of observation was not a reflection of a daily mathematics routine but rather cooperation with the expectations of an intervention programme. If grouping is not based on accurate measurement of developmental levels, it undermines the potential purpose of differentiated instruction.

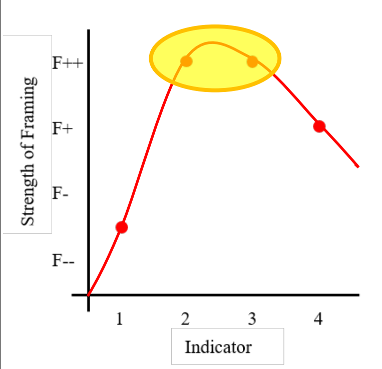
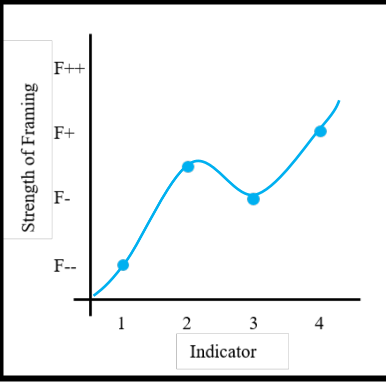
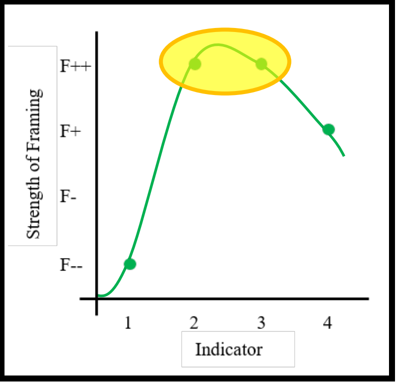
6.2.2 Control relations (framing)

"Within the Bernstein schema, all aspects of pedagogy - pace, selection, sequence, the teacher student relation – are related to or derive from the evaluative criteria (i.e. what is to be transmitted and acquired)" (Hoadley, 2017, p. 17). With the suggestion of a mixed pedagogy, Morais, Neves and Pires (2004) argue primarily for the explication of evaluative criteria. For explication to be successful, it requires open communication relations and if learners are to be explicit in their productions, it should be through interaction with others. Framing over evaluative criteria should therefore be strong while hierarchical rules remain weak. With explication and elaboration also comes weakened framing over pacing in the allocation of time for active engagement in investigative tasks.

Of most significance to this study is a pattern revealed in the framing of the evaluative criteria. While Morais, Neves and Pires (2004) argue for strong and static framing over evaluative criteria throughout, analysis in this study reveals shifting degrees of control over framing as tasks evolve. This again varies according to the type of task and the degree of mediation required. In counting and manipulating number tasks, framing over evaluative criteria is for the most part strong, where the intention and required skill of each task is clear. Most evident however, is a definite trajectory in problem-solving tasks. This is apparent with Teacher A and Teacher C where there is a deliberate pattern or shift in control relations between the teacher and learners, creating the opportunity for mediation and the potential for learning. Aligned with Morais, Neves and Pires' study (2004) is weakened framing over hierarchical rules and pacing in investigative tasks, "creating a context where children can question, discuss and share ideas" (Morais, 2002, p. 561). By asking questions, facilitating discussion and allowing structured communication within the group, mediation is both possible and encouraged.

Through evidence in the data sample, framing varies within problem-solving tasks to produce explicit evaluative criteria. It is the form of content which drives the framing of the pedagogic structure. While all three of the teachers begin and end at similar points in the trajectory, Teacher A and Teacher C take a different path to Teacher B. Teacher A and Teacher C demonstrate an extended form of mediation while Teacher B focuses more attention on the solution to the task, demonstrating a more restricted form.

Table 6.1 A trajectory and platform for mediation within evaluative criteria

Evaluative criteria		
Teacher A	Teacher B	Teacher C
		

In the introduction to a task (Indicator 1), weak control in evaluative criteria is evident across all three of the teachers. Each of the teachers allow learners the opportunity to solve problems without a method prescribed, encouraging them to make sense of the situation and opening the possibility for a range of strategies.

It is in selecting learners to show their strategies for solving the task (Indicator 2) however, where control relations shift significantly. While all three teachers select at least one strategy to be shared with the group, Teacher B does not make a deliberate selection, nor does she check to see whether the solution is correct before sharing with the group.

In the kinds of verbal answers required of learners (Indicator 3), control of framing remains strong for Teacher A and Teacher C. This is the point at which mediation is most pronounced. Both teachers prompt learners with questions and require elaborated reasoning around strategies. By encouraging learners to share ideas with one another, it allows them to verbalise their thought processes through a language that makes sense and internalise mathematical thinking. The other learners in the group are also included in the process, providing the opportunity to see where they may have gone wrong or clarifying misunderstanding. These teachers also sequence the order in which the strategies are revealed to the group with the intention of showing a progression from the least to the most efficient of strategies. It is through this deliberate mediational process that Teacher A and Teacher C intend to reveal different approaches while at the same time guiding learners towards higher levels of efficiency.
For Teacher B however, the mediational process is restricted. Teacher B places more importance on solutions to the task rather than seeking understanding and discussion around strategies used. Interaction and explication are limited, and the teacher is eager to move to the next task.

In concluding the task (Indicator 4), maintaining a slightly lower degree of control, Teacher A and Teacher C implicitly reveal the mathematical skill most desirable while at the same time reminding learners that there are different approaches available. This also suggests the subtle unmasking of an invisible pedagogy. While allowing learners to develop their own strategies and engage in discussion around different approaches, the teacher ultimately points to the method most desirable. While Teacher B also strengthens control, she explicates the correct solution rather than focusing on reasoning or methodologies for achieving the solution.

It can be argued then that it is at these two points of selecting strategies (Indicator 2) and verbally communicating responses (Indicator 3) where Teacher A and Teacher C provide the platform and possibility for extended mediation. Through this mixed pedagogy, Teacher A and Teacher C create a carefully orchestrated process of mediation which actively engages the learners and increases the potential for learning. Teacher B restricts this process of mediation.

6.3 The role of the teacher

"An important condition of children's success in complex cognitive competences is the (scientific) competence of teachers: their knowledge proficiency and command of the investigative competences to be developed" (Morais, Neves and Pires, 2004, p. 84). Through a differentiated approach to teaching and learning, one begins to realise that much of the efficacy is dependent on the teacher. While a comprehensive knowledge of mathematics is essential, one cannot discount the role of the teacher in the mediation process through which mathematical knowledge is acquired.

First is the case of *time and effort*. Naturally, teaching and learning differentially requires both time and an increased amount of effort. It is perhaps easier to plan a lesson for the whole class than it is to prepare three differentiated sets of lessons for a single day. While the time allocation for the mathematics lesson may be the same, the teacher is now expected to complete multiple micro-lessons within the same amount of time. Realistically however, "co-regulation between a teacher and twenty-some students with varying needs and competencies is highly complex in whole-class instruction" (Meyer & Turner, 2002, p. 19). Through constructive planning, the teacher has the capacity to teach mathematics at the correct levels of understanding and better guide learners towards higher level thinking.

Second, is the ability or willingness to relinquish *control*. Differentiated instruction "requires use of open and discursive teaching approaches to which many teachers have a natural resistance; such methods take more time and reduce the apparent control of the teacher" (Swan, 2002, p. 150). While the 'chalk and talk' style of teaching requires less planning, it also allows the teacher the control to avoid questions that may waste time or those which she may not be equipped to answer. It also means that the teacher can 'cover' the necessary curriculum content without disruption or delay. Differentiated teaching relinquishes this control and allows a space for interaction and discussion. The teacher can gauge levels of understanding, build on learners reasoning skills and lead them towards higher levels of competency.

Finally, one needs to consider the teachers *mathematical knowledge* for teaching and learning. Effective teaching is not just about classroom management but rather a management of ideas in identifying 'what' is taught and 'how' such knowledge is presented and transformed to learners in ways comprehendible and meaningful (Shulman, 1986 and 1987). One needs to consider whether the teacher has the fluency and flexibility to facilitate transmission and acquisition of such a process. Shabani argues that while the ZPD certainly applies to learners, surely the same should be applicable to adults, or teachers in specific. Following the idea of 'lifelong professional learning', is the opportunity to facilitate teachers by strengthening their mathematical knowledge.

6.4 Conclusion

> "Different learners' existing understandings meet at the trajectory of the mathematical idea being dealt with at different points. Skilful teacher mediation needs to recognise these different points and work to move these points forward in the context of the classroom" (Venkat, 2013, p. 31).

In this thesis, I have drawn on existing research in the endeavours to define, justify and select differentiated instruction methodologies appropriate to early-grade mathematics. Driven by Hasan who makes connections between Vygotsky (1978) and Bernstein (1990), I place mediation as a central feature to differentiated instruction and focus on the interactional relationship between teacher and taught. By combining Vygotsky's *socio-cultural theory* with Bernstein's framing of *pedagogic discourse*, I investigate how mediation is affected by grouping and control relations in differentiated teaching and learning and how it functions in the development of mathematical understanding.

Using Bernstein's code theory, I have "assigned values, in terms of framing, to the discursive rules of pedagogic practice: the selection, sequencing, pacing and evaluative criteria and hierarchical rules of educational knowledge" (Hoadley, 2005, p. 90). With Bernstein's theory of *pedagogic discourse* as a framework through which mediation can be measured, control relations between teacher and learner reveal subtle differences in differentiating pedagogies and processes of mediation. The analysis challenges the often static coding and characterisations of pedagogy in relation to classification and framing values, showing rather variation and patterning within tasks of framing to produce different pedagogies. In a sense, there is a mix within the mixed pedagogy as different tasks are played out.

While this study provides only a micro lens into differentiated instruction in early-grade mathematics, further research would usefully explore these instances and patterns by relating pedagogy to differential learner outcomes.

References

Aunio, P. & Räsänen, P., 2015. Core numerical skills for learning mathematics in children aged five to eight years - a working model for educators. *European Early Childhood Education Research Journal,* Volume 24, pp. 684-704.

Ball, D., Hill, H., & Bass, H., 2005. Knowing mathematics for teaching. *American Educator*, pp. 14-46.

Bartolini, M. & Mariotti, M., 2008. Semiotic mediation in the mathematics classroom: Artefacts and signs after a Vygotskian perspective. In: *Handbook of International Research in Mathematics Education.* s.l.:s.n., pp. 746-783.

Bernstein, B., 1975. On the classification and framing of educational knowledge. In: *Class, codes and control, volume 3.* London: Routledge and Kegan Paul, pp. 77-106.

Bernstein, B., 1990. *Class, codes and control. Volume IV: The structuring of pedagogic discourse.* London and New York: Routledge.

Brombacher, A., 2011. *The NumberSense Mathematics Programme.* [Online] Available at: https://www.numbersense.co.za/ [Accessed 2019].

Brombacher, A., 2018. *Diagnostic assessment tool.* Cape Town: The NumberSense Mathematics Programme.

Carpenter, T., Fennema, E. & Franke, M., 1996. Cognitively guided instruction: A knowledge base for reform in primary mathematics instruction. *The Elementary School Journal,* Volume 97, pp. 3-20.

Good, T. & Marshall, S., 1984. Do students learn more in heterogeneous or homogeneous groups?. In: *The Social Context of Instruction.* London: Academic Press, pp. 15-37.

Goos, M., 2004. Learning mathematics in a classroom community of inquiry. *Journal for Research in Mathematics Education,* Volume 35, pp. 258-291.

Goos, M., Renshaw, D. & Galbraith, P., 2002. Socially mediated metacognition: Creating collaborative zones of proximal development in small group problem solving. *Educational Studies in Mathematics,* pp. 193-223.

Hasan, R., 2002. Semiotic mediation and mental development in pluralistic societies: Some implications for tomorrow's schooling. In: *Learning for life in the 21st century.* Oxford: Blackwell Publishing, pp. 112-126.

Hasan, R., 2004. The concept of semiotic mediation: Perspectives from Bernstein's sociology. In: J. Muller, B. Davies & A. Morais, eds. *Reading Bernstein, Researching Bernstein.* London and New York: RoutledgeFalmer, pp. 30-41.

Hasan, R., 2005. Semiotic mediation, language and society: Three exotripic theories - Vygotsky, Halliday and Bernstein. In: J. Webster, ed. *The collected works of Ruqaiya Hasan.* Cape Town: Equinox Publishers, pp. 130-156.

Hennessy, S. et al., 2016. Developing a coding scheme for analysing classroom dialogue across educational contexts. *Learning, culture and social interaction,* Volume 9, pp. 16-44.

Hermkes, R., Mach, H. & Minnameier, G., 2018. Interaction-based coding of scaffolding processes. *Learning and Instruction,* Volume 54, pp. 147-155.

Hoadley, U., 2003. Time to learn: Pacing and the external framing of teachers work. *Journal of Education for Teaching,* pp. 265-277.

Hoadley, U., 2005. Social class, pedagogy and the specialization of voice in four South African primary schools. Unpublished PhD thesis, Cape Town, University of Cape Town.

Hoadley, U., 2005. Social class and pedagogy: A model for the investigation of pedagogic variation. *British Journal of Sociology of Education,* pp. 63-78.

Hoadley, U., 2006. Analysing pedagogy: The problem of framing. *Journal of Education, no. 40,* pp. 15-34.

Hoadley, U., 2017. Learning to fly: Pedagogy in the foundation phase in the context of the CAPS reform. *Journal of Education,* Volume 67, pp. 13-38.

Hoadley, U. & Muller, J., 2010. Codes, pedagogy and knowledge: Advances in Bernsteinian sociology of education. In: M. Apple, S. Ball & L. Gandin, eds. *The Routledge International Handbook of the Sociology of Education.* London and New York: Routledge, pp. 69-78.

Kilpatrick, J., Swafford, J. & Findell, B., 2001. *Adding it up: Helping children learn mathematics.* Washington, DC: National Academy Press.

Lin, A. & Small, M., 2010. *More good questions: Great ways to differentiate mathematics.* New York: Teachers College Press.

Linchevski, L. & Kutscher, B., 1998. Tell me with whom you're learning, and I'll tell you how much you've learned: Mixed ability versus same-ability grouping in mathematics. *Journal for Research in Mathematics Education,* Volume 29, pp. 533-554.

Meyer, D. & Turner, J., 2002. Using instructional discourse analysis to study the scaffolding of student self-regulation. *Educational Psychologist,* Volume 37, pp. 17-25.

Morais, A., 2002. Basil Bernstein at the micro level of the classroom. *British Journal of Sociology of Education,* pp. 559-569.

Morais, A., Neves, I. & Pires, D., 2004. The what and how of teaching and learning: Going deeper into sociological analysis and intervention. In: J. Muller, B. Davies & A. Morais, eds. *Reading Bernstein, Researching Bernstein.* New York and London: Routledge Falmer, pp. 75-90.

Muller, J. & Young, M., 2013. On the power of powerful knowledge. *Review of Education,* pp. 229-250.

Pausigere, P., 2016. On Bernstein's sociology of pedagogy and how it can inform the pedagogic realisation of poor and working-class children in South African Primary Maths Education. *Educational Research for Social Change ,* Volume 5, pp. 41-53.

Pedro, E., 1981. *Social stratification and classroom discourse: A sociolinguistic analysis of classroom practice.* Stockholm: Stockholm Institute of Education, Department of Educational Research.

Schmitt, E., 2009. *Pedagogic scaffolding practices of primary mathematics teachers.* Unpublished Masters thesis, Cape Town, University of Cape Town.

Schoenfeld, A., 2017. Video analyses for research and professional development*: The teaching for robust understanding (TRU) framework,* Berkeley: Springer.

Shabani, K., 2010. Vygotsky's zone of proxiaml development: Instruction implications and teachers' professional development. *English Language Teaching,* Volume 3, pp. 237-248.

Shabani, K., 2016. Applications of Vygotsky's sociocultural approach for teachers' professional development. *Cogent Education,* Volume 3.

Shotter, J., 1993. Vygotsky: The social negotiation of semiotic mediation. *New ideas in Psychology,* Volume 11, pp. 61-75.

Shotter, J. & Lock, A., 2012. Sixth sense, second nature, and other cultural ways of making sense of our surroundings: Vygotsky, Bernstein and the languaged body. In: H. Daniels, ed. *Vygotsky and Sociology.* London and New York: Routledge, pp. 61-79.

Singh, H., 2014. Differentiating classroom instruction to cater learners of different styles. *Indian Journal of Applied Research,* Volume 3, pp. 58-60.

Singh, P., 2002. Pedagogising Knowledge: Bernstein's theory of the pedagogic device. *British Journal of Sociology of Education.*

Slavin, R. & Karweit, N., 1985. Effects of whole class, ability grouped, and individualised instruction on mathematics achievement. *American Educational Research Journal,* Volume 22, pp. 351-367.

Sørensen, A. & Hallinan, M., 1986. Effects of ability grouping on growth in academic achievement. *American Educational Research Journal,* Volume 23, pp. 519-542.

Subban, P., 2006. Differentiated instruction: A research basis. *International Education Journal,* Volume 7, pp. 935-947.

Swan, M., 2002. Dealing with misconceptions in mathematics. In: P. Gates, ed. *Issues in Mathematics Teaching.* s.l.: Routledge, pp. 147-165.

Tomlinson, C., 2004. Sharing responsibility for differentiated instruction. *Education Resources Information Centre,* Volume 26, pp. 188-189.

van de Pol, J., Volman, M. & Beishuizen, J., 2010. Scaffolding in teacher-student interaction: A decade of research. *Educational Psychology Review,* Volume 22, pp. 271-296.

Venkat, H., 2013. Using temporal range to theorize early number teaching in South Africa. Volume 33, pp. 31-37.

Venkat, H. & Naidoo, D., 2012. Analysing coherence for conceptual learning in a Grade 2 numeracy lesson. *Education as Change,* Volume 16, pp. 21-33.

Vygotsky, L., 1978. *Mind in Society: Development of higher psychological processes.* Cambridge, MA: Harvard University Press.

Yin, R., 1984. *Case Study Research: Design and Methods.* California: Sage Publications.

Young, M., 2009. Education, globalisation and the 'voice of knowledge'. *Journal of Education and Work,* pp. 193-204.

Appendix

Coding scheme for the framing of pedagogic practice

PEDAGOGIC STRUCTURE

Discursive rule **SELECTION** (F^{+-})
The extent to which the teacher and learners have control over the selection of instructional knowledge

1. In the exposition to a task and in doing activities	**F^{++}**	**F^{+}**	**F^{-}**	**F^{--}**
	Always or almost always controlled by the teacher	**Mostly controlled by the teacher**	**Learners have some control**	**Learners have substantial control**
	The selection of tasks, activities and knowledge in the group is always or almost always determined by the teacher. Learners are rarely able to disrupt the selection to suit their own needs. Their interjections are generally dismissed or ignored, or they are not seen to make any interjections.	The selection of tasks, activities and knowledge in the group is determined by the teacher most of the time. On few occasions, selection is varied according to learners' interjections, productions and understanding.	Learners have the opportunity to vary the selection of tasks, activities and knowledge some of the time. Some learner suggestions are accepted, or the teacher alters selection according to learners' interjections, productions and understanding.	Learners often make decisions around the selection of tasks, activities and knowledge in the group. The teacher alters the selection according to learners' interjections, productions and understanding.

Discursive rule **SEQUENCING** (F^{+-})
The extent to which the teacher and learners have control over the sequencing of instructional knowledge

2. In the course of the group session	**F^{++}**	**F^{+}**	**F^{-}**	**F^{--}**
	Always or almost always controlled by the teacher	**Mostly controlled by the teacher**	**Learners have some control**	**Learners have substantial control**
	The teacher always or almost always determines the sequence of transmission of knowledge in the group. Any interjections potentially disturbing the order of learning are dismissed or ignored.	The teacher mostly determines the sequence of transmission of knowledge in the group.	Learners sometimes make decisions around the sequence of tasks and activities in the group. They are at times given options regarding the order in which to do things.	Learners have the opportunity to vary the sequence of the transmission often. The teacher often responds to learners' interventions by varying the sequence of the learning.

Discursive rule **PACE** (F^{+-})
The extent to which the teacher and learners have control over the pacing of instructional knowledge

3. In the learners doing activities / tasks	F^{++}	F^{+}	F^{-}	F^{--}
	Always or almost always controlled by the teacher	**Mostly controlled by the teacher**	**Learners have some control**	**Learners have substantial control**
	The pace at which learners work through tasks is always or almost always strictly controlled by the teacher. Injunctions to 'hurry up' or 'work slowly' are frequent, and the teacher does not vary the pace according to learners' productions. The teacher always or mostly defers or ignores learners' questions and interjections, or learners make no interjections.	The pace at which learners work through tasks is mostly determined by the teacher. Time is mentioned quite often and on occasion the length of an activity is stipulated beforehand. The teacher accepts few learner interventions or questions. She answers questions briefly and moves on. Occasionally, the teacher varies the pace in response to learners' productions and levels of understanding.	Learners work at their own pace. The teacher exercises some control over pace but remains open to its variation. The teacher accepts some learner interventions and questions. She pauses the group activity briefly to make sure that all learners are ready to move on before doing so.	Learners work at their own pace. The teacher places no pressure on them to finish in a stipulated period. She may give them opportunities to 'catch up'. The teacher accepts most or all learners' interventions and questions and discussion may be extended or deviate as a result. Learners decide when they are ready to move on to the next task.

Discursive rule **EVALUATIVE RULES** (F^{+-})
The extent to which the teacher and learners have control over the evaluative criteria of the instructional knowledge pertaining to the meaning of concepts and principles and their appropriate realisation

4. In the introduction / explanation / exposition to a task	F^{++}	F^{+}	F^{-}	F^{--}
	Always or almost always controlled by the teacher	**Mostly controlled by the teacher**	**Learners have some control**	**Learners have substantial control**
	The teacher always or almost always makes the evaluative rules available through exposition. The teacher explicitly defines and explains the meaning of concepts, addressing key aspects of the knowledge or operation under discussion. She is clear as to how a task should be completed.	In most cases, the teacher makes the evaluative rules available in an explicit and clear manner. The requirements for the successful production of a task are generally clear, although there may be some aspects that remain implicit.	The concepts and principles being addressed in the exposition are sometimes unclear. Attempts are made in indicating the requirements for the successful production of a task, but these are often unclear or not articulated. There is some ambiguity as to how the task should be completed.	Generally, the teacher does not draw out the knowledge principles in her exposition. Very little or no attempt is made in indicating the requirements for the successful production of a task. Learners are unclear as to how to proceed or continue in any manner they choose.

5. In selecting learners to show their solutions to a task or activity	F^{++}	F^{+}	F^{-}	F^{--}
	Predominantly high level of selection	**Some high level of selection**	**Mostly low level of selection**	**Predominantly low level of selection**
	The teacher always selects a range of successful strategies from the learners in the group.	The teacher selects one or more successful strategies from learners in the group.	The teacher selects any strategies from learners in the group.	The teacher selects any or no strategies from learners in the group.

6. In the kinds of verbal answers required of learners	F^{++}	F^{+}	F^{--}	F^{--}
	Evaluative criteria very clear and explicit	**Evaluative criteria quite clear and explicit**	**Evaluative criteria quite unclear and implicit**	**Evaluative criteria very unclear and implicit**
	Learners are always or almost always required to give reasons for their answers. They may be asked to draw out a more general principle to support, clarify or modify their answer. In incorrect responses, the teacher shows why the answer is incorrect. The teacher often elaborates on a correct answer.	Learners are often required to give reasons for their answers. They are sometimes asked to clarify or modify their answers. In incorrect responses, the teacher often shows why the answer is incorrect. The teacher often elaborates on a correct answer.	Learners are on a few occasions required to give reasons for their answers. In incorrect responses, the teacher sometimes shows why the answer is incorrect. The teacher does not elaborate on a correct answer.	The teacher looks only for yes / no answers, or for learners to repeat what she has just said. Incorrect answers are generally ignored or the reasons for them are not sought. Correct answers are accepted and may be praised but are not elaborated on.

7. In concluding the task / activity	F^{++}	F^{+}	F^{--}	F^{--}
	Evaluative criteria very clear and explicit	**Evaluative criteria quite clear and explicit**	**Evaluative criteria quite unclear and implicit**	**Evaluative criteria very unclear and implicit**
	The teacher makes specific comments around what constitutes an appropriate production. There is rigorous evaluation of learners' productions. She gives examples of both success and failure in the task and points to individual performances.	The teacher comments on what constitutes a successful production, directed at the group as a whole. Success or failure is indicated, and the teacher gives examples of what constitutes an appropriate production.	Learners work is monitored but with little or no comment as to what constitutes an appropriate production. The teacher provides little or no indication of success or failure in their attempts.	The teacher looks at productions but makes little or no comment. Learners are not given access to the criteria for success or failure in their productions. Correct solutions are not displayed for learners.

Hierarchical rule **TEACHER – LEARNER** (F^{+-})

The extent to which the teacher and learners have control over the order, character and manner of the conduct of learners in the relation between teacher and learner

8. In facilitating discussion in the group	F^{++}	F^{+}	F^{-}	F^{--}
	Positional	**Mostly positional**	**Personal or positional**	**Mostly personal**
	The teacher always directs the discussion and provides no opportunity for the learners to show their individual thought processes.	The teacher mostly directs the discussion and allows little opportunity for individual learners to express their thought processes.	The teacher engages with some learners in the group and attempts to understand some thought processes.	The teacher allows learners to speak freely and attempts to understand learners thought processes.

9. In presenting solutions to the group	F^{++}	F^{+}	F^{-}	F^{--}
	Positional	**Mostly positional**	**Personal or positional**	**Mostly personal**
	Learners demonstrate and explain their solutions only to the teacher. The other learners in the group do not engage or participate in the presentation.	Learners demonstrate and explain their solutions mostly to the teacher. Most of the other learners in the group do not engage or participate in the presentation.	Learners demonstrate and explain their solutions to the teacher and those listening in the group. The other learners in the group sometimes engage or participate in the presentation.	Learners demonstrate and explain their solutions to the whole group. The other learners in the group are actively engaged and participate in the presentation.

10. In communication relations between the teacher and the learners on the mat	F^{++}	F^{+}	F^{-}	F^{--}
	Positional	**Mostly positional**	**Mostly personal**	**Personal**
	The teacher explicitly closes teacher-learner communication relations. Learners participate in teacher-learner interactions only when invited to do so through the teacher's questioning.	The teacher limits teacher-learner communication relations. Learners respond to questions or discussion initiated by the teacher but seldom make their own comments or interjections.	Teacher-learner communication relations are sometimes encouraged. The teacher elicits interaction by prompting learners with questions and making comments around their productions. The teacher responds to questions and offers assistance where needed.	Open teacher-learner communication relations are always promoted. Learners initiate interaction with the teacher of their own accord by commenting, interjecting, asking questions about mathematics or requesting assistance from the teacher.

11. In the rapport between the teacher and learners on the mat	**F^{++}**	**F^{+}**	**F^{-}**	**F^{--}**
	Positional	**Mostly positional**	**Mostly personal**	**Personal**
	The teacher does not display any form of friendliness or openness. The teacher offers no words of praise during the mat routine. She does not promote peer commendation.	The teacher seldom interacts with learners in an informal or friendly manner. The teacher rarely offers any word of praise or recognition during the mat routine, nor does she encourage it from the other learners.	The teacher is often open and friendly and praises learners for their efforts or behaviour. The teacher may also encourage other learners to commend a successful learner.	The teacher is informal, open and friendly towards the learners. She praises learners for their success or efforts and promotes motivation from other learners in the group.

Coding scheme for Teacher A

Group	Task	Selection	Sequence	Pace	Evaluative rules	Hierarchical rules
1	**Counting**	F^{++}	F^{++}	F^{+}	F^{+}	F^{--}
					F^{--}	F^{-}
					F^{+}	F^{-}
					F^{+}	F^{-}
	Manipulating number	F^{++}	F^{++}	F^{-}	F^{-}	F^{-}
					F^{-}	F^{-}
					F^{+}	F^{-}
					F^{-}	F^{-}
	Problem solving	F^{++}	F^{+}	F^{-}	F^{--}	F^{--}
					F^{++}	F^{--}
					F^{++}	F^{--}
					F^{++}	F^{--}
2	**Problem solving**	F^{++}	F^{++}	F^{-}	F^{-}	F^{--}
					F^{+}	F^{-}
					F^{++}	F^{--}
					F^{+}	F^{--}
	Counting	F^{++}	F^{++}	F^{+}	F^{+}	F^{--}
					F^{--}	F^{-}
					F^{+}	F^{--}
					F^{+}	F^{-}
	Manipulating number	F^{++}	F^{++}	F^{+}	F^{+}	F^{-}
					F^{-}	F^{-}
					F^{+}	F^{-}
					F^{+}	F^{-}
3	**Problem solving**	F^{++}	F^{++}	F^{-}	F^{--}	F^{--}
					F^{++}	F^{--}
					F^{++}	F^{--}
					F^{+}	F^{--}
	Problem solving	F^{++}	F^{++}	F^{-}	F^{-}	F^{--}
					F^{++}	F^{--}
					F^{++}	F^{--}
					F^{+}	F^{--}
	Manipulating number	F^{++}	F^{+}	F^{+}	F^{+}	F^{+}
					F^{-}	F^{+}
					F^{--}	F^{+}
					F^{+}	F^{-}

Coding scheme for Teacher B

Group	Task	Selection	Sequence	Pace	Evaluative rules	Hierarchical rules
1	Counting	-	-	-	-	-
					-	-
					-	-
					-	-
	Manipulating number	F^{++}	F^{++}	F^{+}	F^{+}	F^{+}
					F^{-}	F^{+}
					F^{--}	F^{+}
					F^{+}	F^{-}
	Problem solving	F^{++}	F^{+}	F^{-}	F^{--}	F^{-}
					F^{+}	F^{+}
					F^{-}	F^{+}
					F^{+}	F^{-}
	Problem solving	F^{++}	F^{++}	F^{+}	F^{--}	F^{+}
					F^{-}	F^{-}
					F^{-}	F^{-}
					F^{-}	F^{-}
2	Manipulating number	F^{++}	F^{++}	F^{+}	F^{-}	F^{-}
					F^{+}	F^{-}
					F^{-}	F^{-}
					F^{+}	F^{--}
	Manipulating number	F^{++}	F^{+}	F^{+}	F^{-}	F^{-}
					F^{-}	F^{-}
					F^{+}	F^{-}
					F^{+}	F^{-}
	Manipulating number	F^{+}	F^{+}	F^{+}	F^{+}	F^{-}
					F^{-}	F^{-}
					F^{+}	F^{--}
					F^{+}	F^{-}
3		-	-	-	-	-
					-	
					-	-
					-	
		-	-	-	-	-
					-	
					-	-
					-	
		-	-	-	-	-
					-	
					-	-
					-	

Coding scheme for Teacher C

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<tr><td>F^{-}</td><td rowspan="2">F^{--}</td></tr>
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